The Art of Timing: Managing Sudden Stop Risk in Corporate Credit Markets*

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Abstract

High yield firms nowadays almost exclusively issue bonds that are callable. We construct a new measure of option moneyness and show that firms aggressively exercise the interest rate and spread option implicit in these contracts. Controlling for moneyness, firms frequently prepay bonds and issue new debt if rollover risk is high. We develop and estimate a structural model to quantify the costs and benefits of dynamically managing this risk. The ability to use callable debt almost entirely dissipates dead-weight losses from rollover risk. Creditor-shareholder conflicts reduce the effectiveness of this dynamic hedging strategy for highly levered firms.

Keywords: Corporate Leverage, Call Provisions, Rollover-risk, Agency Conflicts.

JEL codes: G32

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1 Introduction

The corporate bond market is an essential source of capital for large firms in the US, and its proper functioning ensures that firms can continue to finance their investments, make payroll, and rollover their outstanding debt. As the early days of the Covid crisis illustrate, stress in bond markets sometimes leads to sudden stops, exposing the US corporate sector — and in particular, non-investment grade firms — to the risk of failure when debt obligations come due and investors are on strike. In this paper, we argue that high-yield firms proactively manage the term structure of their debt maturities through the issuance of callable bonds and the exercise of these call options in order to mitigate their exposure to credit market shutdown risk.

We document that high-yield firms, for the past 25 years, have systematically relied on debt with various call protection structures in order to access the bond market. We show that these firms, driven by an interest cost savings motive, have aggressively exercised the interest rate and credit spread option embedded in these instruments over this time period of declining long term rates. However, a second consideration has driven firms' early refinancing behavior: a rollover risk management motive. We decompose these two distinct economic forces in the data, and compute, through the lens of a model, the cost incurred by these firms when hedging against credit market shutdown risk. While costly, this dynamic debt maturity management enhances shareholders' and creditors' value, compared to a counterfactual environment in which high yield firms could only access the bond market via non-callable debt.

While we are not the first to analyze firms' debt rollover risk, our study emphasizes the difficulty to separate, in the data, early refinancing behavior driven by that motive vs. that driven by the traditional interest cost savings' incentives. We address this challenge by building a model-free empirical measure of moneyness for call options embedded in high yield corporate bonds, and relate it to option exercise probabilities. This empirical exercise not only shows that high-yield firms exercise the interest rate and spread options embedded in callable debt actively, but also that, controlling for option moneyness, rollover risk is an important driver of prepayment behavior.

Our paper is also the first, to our knowledge, to formalize these distinct economic mechanisms — interest cost savings and rollover risk-management — through a theory that (i) we can confront to the data and that (ii) allows us to compute counterfactuals not available via reduced form empirics. Our theory allows us to uncover a new form of creditor-shareholder agency conflicts: as the cost of rollover risk management is borne by shareholder but the benefit partially accrues to debt holders, greater corporate leverage distorts firms' management optimal decisions and exacerbates dead-weight losses, leading to agency issues similar to the widely-studied debt overhang channel.

We begin our study by documenting the type of callable bond debt issued by high-yield firms since 2000. These firms have gradually shifted from issuing fixed-price callable bonds to issuing *hybrid* bonds — callable at prices (i) varying with the level of interest rates early on in the life of the bond, and (ii) fixed later in the bond's life. Both types of securities exhibit weak prepayment protections, in stark contrast to those sold by investment-grade corporate borrowers. Thus, the interest rate and credit spread optionality embedded in high-yield bonds allows their issuers to strategically refinance for interest cost savings motives.

We then show, via survival analysis, that high-yield firms — for the last 25 years — have aggressively refinanced their callable bonds as long term rates have drifted downward and with temporary falls in credit spreads. As a result, while the contractual life of high-yield debt has averaged 9 years, its effective life has been 6 years only. Aggregate bond prepayment rates over that time period have averaged 12.1% on a annual basis and have exhibited significant time-series variation, falling during the GFC, the credit market stress period of 2015-2016, and at the beginning of the Covid crisis. These credit supply shocks have emphasized the fragile nature of the high yield bond market and, we argue, incentivize firms to early refinance strategically as a way to mitigate rollover risk.

In order to disentangle firms' interest cost savings motive from this rollover risk management channel, we build a measure of moneyness of the interest rate and spread option embedded in callable corporate debt. To do this, we need to compare the strike price of these call options — the contractual call price at which a firm can early retire its bond — with the spot price. Since high-yield firms rarely issue bullet non-callable debt, we cannot observe directly this spot price; instead, we replicate synthetically the price of non-callable debt of the issuer by combining a risk-free bullet fixed rate bond with a (non-callable) CDS contract, carefully taking into account the (negative) basis between bonds and CDS. Our measure of moneyness is tightly related to call option exercise probabilities; changes in market conditions (or the passage of time) that move moneyness from negative to positive territory are associated with 10 p.p. increase in *monthly* call option exercise probability. Not surprisingly, this estimate suggests that firms are much more aggressive in exercising their call options relative to US households, who exhibit sluggish behavior when refinancing their fixed-rate prepayable mortgage debt.

Armed with our measure of moneyness, we can hold constant the value of the interest rate and credit spread option (and thus the interest cost savings incentives), and isolate the rollover risk management motive when studying firms' early refinancing behavior. With these controls, we show that firms with a greater amount of debt coming due in the next 2 years have a greater probability of early refinancing their callable bonds. Even when the rate and spread option is out of the money, firms do not hesitate to pay costly call premium to bond holders in order to prepay their outstanding debt and push their debt maturities further into the future.

Our reduced form empirical analysis helps emphasize the separate motives for firms' strategic call options' exercise. But it does not allow us to estimate the cost borne by firms facing sudden stop risk, or the cost incurred by these firms (via call premium payments to creditors) in order to mitigate this risk. To help shed light on these questions, we build a structural model of a firm that finances itself with callable bullet-maturity debt, in an environment where credit supply is uncertain: either capital market are open, and a firm that needs to refinance is able to do so, or capital markets are in a sudden stop state, in which case a firm whose bonds are maturing is unable to rollover its debt and must default. In that simple model, strategic refinancing decisions are purely driven by rollover risk hedging considerations: firms weigh the benefit of early refinancing — the ability to push debt maturity further into the future while markets are still open — with its costs — the call premia that need to be paid to bond investors. We characterize completely the equilibrium of our model, showing that firms follow a cutoff rule: refinance as soon as the time-to-maturity of the outstanding debt falls below an endogenous threshold, at which point the marginal equity benefit of waiting slightly longer is equalized with the marginal change in call premium cost.

Our simple model allows us to uncover a new source of creditor-shareholder agency conflicts: since the cost of early refinancing is borne by shareholder but the benefits partially accrue to the firm's creditors, firm's managers (acting on behalf of shareholders) delay their refinancing decisions — in sharp contrast to the actions that would be taken by managers maximizing firm value. This agency conflict naturally affects enterprise value: while the dynamic call option exercise strategy of firms' managers mitigates dead-weight losses from defaults due to credit market shutdowns, greater corporate leverage distorts managers' refinancing decisions, increases the frequency of defaults and lowers enterprise value.

We then expand our simple model to introduce stochastic interest rates and default intensities, so that callable debt prepayment decisions are now jointly driven by (a) interest cost savings and (b) rollover risk management motives. We estimate a standard model for interest rates and default intensities, calibrate the remaining parameters, solve our model, confront its predictions to their data counterparts, and study the impact of sudden stop risk on firms' behavior and claims' valuation. Our computations suggest that firms' ability to use callable debt — and the associated optimal prepayment and early refinancing strategy followed by firm's managers — almost entirely dissipates the dead-weight losses arising from default due to sudden stops. In particular, when sudden stop risk is moderate, firms limited to using non-callable debt have enterprise value on average 4% lower than firms who are able to dynamically manage the maturity structure of their debt via call options. Relative to a counterfactual environment without rollover risk, firm shareholders spend on average 20bps p.a. (of debt balance) in additional call premia in order to hedge sudden stop risk, and prepay their debt at 20% higher frequency. Greater corporate leverage reduces firms' effectiveness in sudden stop risk hedging, a natural consequence of the creditor-shareholders agency conflict we uncover in this paper.

The remainder of the paper is structured as follows: Section 2 discusses the related literature. Section 3 lists the various possible motives for firms to prepay their debt early. Section 4 describes the data and our sample selection procedure. In Section 5, we study the issuance and prepayment behavior of high-yield firms and discuss our strategy to separately identify interest cost savings and rollover risk management incentives. In Section 6 we introduce our theory of rollover risk, while in Section 7 we take our model to the data and use it to study counterfactuals not available through reduced form empirics. Section 8 concludes.

2 Literature

Our paper relates to the literature on rollover risk. Theoretical contributions include Brunnermeier and Yogo (2009), He and Xiong (2012*a*), He and Xiong (2012*b*), Diamond and He (2014) and He and Milbradt (2016), among others. This literature shows that rollover risk affects credit risk, exacerbates debt overhang, and exposes firms to costly debt restructuring risk. We contribute to this literature by showing how callable debt allows firms to dynamically hedge sudden stop risk and by quantifying the cost and benefits of this risk management strategy. Further, we emphasize a new form of creditor-shareholder agency conflict— since hedging costs are borne by shareholders but some of the benefits are captured by creditors, managers inefficiently delay refinancing decisions.

Empirical studies confirm that rollover risk can affect asset prices and firm performance. Using different data, Valenzuela (2016) and Nagler (2020) both find that the effects of debt market illiquidity on bond spreads is exacerbated by high degrees of firm-level rollover risk. Duval, Hong and Timmer (2019) shows that firms with higher balance-sheet vulnerabilities suffered greater TFP slowdowns post GFC. Almeida et al. (2012) provide evidence that firms whose long-term debt was due during the GFC reduced their investment significantly relative to otherwise similar firms. In a similar spirit, DeFusco, Nathanson and Reher (2023) show that operators of hotels financed by commercial mortgage debt that was maturing during the early months of the Covid pandemic had significantly lower revenues and underinvested, relative to those with debt that was refinanced just before Covid. Many of these studies document that the cost of not being able to rollover their debt can be sizable for firms. It is therefore not surprising that the risk of having to refinance in "bad times" is also frequently cited by CFOs as key determinant of debt maturity choice (Graham and Harvey, 2001).

Firms can actively manage this risk via their financial policies. Harford, Klasa and Maxwell (2014) shows how firms mitigate rollover risk by increasing their cash holdings. Choi, Hackbarth and Zechner (2018) argue that firms increase the dispersion of their debt maturity when they need to rollover outstanding debt. Parise (2018) shows that airlines under the threat of increased competition by new entrants lengthen the maturity of their debt. These studies focus on whether firms' average debt maturity or maturity dispersion changes when rollover risk increases. The literature on firms' use of callable debt as a hedging device for rollover risk is scant. A notable exception is Xu (2018), who also highlights that high-yield firms use call provisions embedded in bond contracts in order to extend the maturity of their debt.

Our study emphasizes the difficulty to separate, in the data, early refinancing behavior driven by the rollover risk management channel vs. that driven by the traditional interest cost savings' incentives. Rather than modulating the contractual maturity of their debt depending on market conditions and specific investment needs, most high-yield firms issue debt with a maturity of 10 years, possibly due to institutional frictions and investor demand. Thus, firms' average debt maturity automatically extends after any refinancing event, irrespective of the prepayment motive. To address this challenge — identifying the link between rollover risk and refinancing via the exercise of bond call options — we propose a new empirical measure of moneyness for call options embedded in corporate bonds, and relate it to prepayment probabilities. This enables us to show that rollover risk is an important driver of prepayment behavior, controlling for any interest cost savings' motives.

Our paper also contributes to the literature on call provisions in bond contracts and debt prepayments. Tewari, Byrd and Ramanlal (2015), Brown and Powers (2020) and Powers (2021) study the issuance of callable bonds, as well as the determinants of call premiums and call protection provisions. In our paper we document recent origination trends in high-yield credit markets — in particular the quasi systematic use of "hybrid" callable debt, a type of instrument combining features of make-whole and fixed-price callable bonds. A related literature analyzes theoretically and empirically different motives for firms to retire their debt early. We discuss this literature in detail in the next section, and highlight the two main motives that will be the focus of our study: the interest cost savings and rollover risk management incentives.

3 Prepayment motives

We briefly review various explanations put forward by the literature for why firms might want to early repay their financial obligations. While we discuss informally the *interest cost savings* and *rollover risk management* motives — the key focus of our paper — we define those concepts precisely, through the lens of a model, in Section 6.

First, prepayments can be driven by the desire to reduce the firm's debt interest expense — the *interest cost savings* channel. This channel operates through slightly different mechanics, depending on whether the debt instrument carries a fixed interest rate — in which case it embeds both an interest rate and credit spread option — or a floating interest rate — in which case it only carries a credit spread option. On the one hand, callable fixed rate bonds (i.e., most high-yield bond debt) provide the issuer with an option to early repay the principal balance (subject to contractually specified prepayment penalties); upon a decline of either long term interest rates or credit spreads (or both), firms might want to prepay and refinance such debt, so as to take advantage of lower bond yields (see, among others, Merton, 1974; Brennan and Schwartz, 1977; Vu, 1986; Mauer, 1993; Longstaff and Tuckman, 1994; Acharya and Carpenter, 2015; Jarrow et al., 2010). On the other hand, floating rate debt (i.e., most bank debt) can almost always be refinanced at minimal costs, and carries an interest rate equal to a benchmark reference rate (usually 3-month LIBOR, or more recently SOFR) plus a fixed spread; thus, it only embeds a credit spread option, as changes in market interest rates are automatically passed-through (Ippolito, Ozdagli and Perez-Orive, 2018).

Second, firms can manage their debt maturity structure and mitigate their exposure to credit supply shocks by early refinancing their term debt — the *rollover risk management* channel. Sudden drops in market liquidity (He and Xiong, 2012*b*) or "market shutdowns" (Brunnermeier and Yogo, 2009) can prevent firms from rolling over their debt or make refinancing very costly. Hence, firms might want to refinance their long-term debt ahead of time if financial markets are currently "open" to manage the uncertainty that credit market conditions might deteriorate near the contractual bond maturity. That is, firms can use prepayments to dynamically manage rollover risk (Xu, 2018).

Third, firms might have an incentive (not) to early refinance their debt if doing so could remove (would add) restrictive covenants — the *covenant switch* channel. This idea is, e.g., formalized in Green (2018) who shows that, all else equal, firms that have been upgraded to investment-grade ("rising stars") refinance earlier in order to remove restrictive high-yield covenants, while firms that have lost their investment-grade rating ("fallen angels") refinance later to avoid adding new restrictions. See also King and Mauer (2000) for similar arguments.

Finally, including call provisions in debt contracts might mitigate conflicts of interest between shareholders and debt holders — the *agency* channel. Bodie and Taggart (1978) argue theoretically that call provisions can help address under-investment problems as firms can re-contract before making investment decisions (and thus avoid that some profitable investment projects that might benefit existing debt holders are not undertaken). Using a similar approach, Barnea, Haugen and Senbet (1980) show theoretically how call provisions can also mitigate agency problems related to risk-shifting and information asymmetry. Both papers suggest that shorter term debt could achieve the same objectives. Empirically, this debate is far from being settled: while Crabbe and Helwege (1994) provides evidence against each of these agency theories, Becker et al. (2022) find evidence that callable debt reduces debt overhang in corporate merger activity.

In what follows, we provide empirical evidence for the role of interest cost savings and rollover risk management motives and then develop a model to quantify the costs spent by firms mitigating credit market shutdown risk via the strategic exercise of debt call options.

4 Data and sample selection

In order to construct our instrument-month panel for our sample period 2000 to 2022, we leverage data from Mergent FISD ("Mergent") and CRSP Compustat Merged ("CCM") via Wharton Research Data Services ("WRDS"). Mergent provides us with the contractual details of the corporate bonds issued by firms in our sample, as well as events that entail a change in the notional balance of these bonds. Specifically, for each bond we track the outstanding balance over time and classify changes into scheduled amortizations and unscheduled principal prepayments, as well as other extraordinary events (e.g., upsizings or defaults). We further collect details on prepayment events such as prepayment method (call, tender offer, open market repurchase) and price at which those events take place (the so-called "action price" in Mergent).

CCM allows us to collect quarterly income statement and balance-sheet related information, and track historical CUSIP changes of the firms in our sample. We rely on TRACE in order to compute corporate bond prices and yield to maturity ("YTM"), and use time series data for interest rate swaps and treasuries from FRED in order to back out corporate bond yield spreads over the relevant benchmark. In later sections, we use CDS data from IHS Markit to calculate prices for counterfactural non-callable instruments.

Our final sample comprises all bonds outstanding by all US firms between 2000 to 2022, excluding:

- firms whose aggregated bond outstanding balance cannot be observed for at least three consecutive years (where the aggregated bond outstanding balance is constructed using information from Mergent);
- firms in industries with SIC codes 4900-4999 and 6000-6999; in other words, we exclude financials and utilities.

Appendix A.2 contains greater details on our sample construction, and various choices we had to make throughout the data cleaning process.[1] Our final sample includes 21,508 bonds — 16,015 investment grade and 5,493 non-investment grade securities, using the instrument-specific credit rating at the time of issuance — issued by 1,853 non-financial US firms. In what follows, "IG" will denote investment grade debt issuers, whereas "HY" will denote non-investment grade debt issuers. While our study is mostly focused on HY firms and their dynamic debt prepayment strategies, when appropriate we contrast our data for HY issuers with the relevant data for IG issuers, to emphasize the special nature of HY credit markets. Table 1 shows various statistics on bond and issuer characteristics for our sample of HY bonds.

5 Prepayment behavior in credit markets

5.1 Call protection at issuance

We partition our bond sample into bonds that are either (i) "non-callable", (ii) "make-whole callable", (iii) "fixed-price callable", or (iv) "hybrids". Our classification resembles that of Powers (2021), with a smaller number of distinct types, for simplicity.

- "Non-callable" bonds cannot be retired early during their contractual life.[2]

[1]For example, as highlighted by Powers (2021), 144A bonds are frequently exchanged for an identical publicly registered bond(s). We follow a procedure similar to Powers (2021) to identify exchanged issues in the data to avoid double-counting of bonds.

[2]Non-callable bonds may include provisions that allow the bond to be called under "special circumstances" (e.g., in the event of a takeover).

Table 1: Descriptive statistics

This table shows summary statistics for our sample of non-investment grade bonds. Panel A reports statistics on instrument and issuer characteristics (at the time of issuance) at the bond-level. Panel B shows summary statistics at the bond-month-level. All variables are defined in detail in Appendix Table A-1. Note that at the bottom of Panel A bonds are classified by their bond-specific call type. The call status reported at the bottom of Panel B can be time varying, i.e., is defined at the instrument-month level. For instance, a fixed-price callable bond can have bond-month observations during with the instrument is not (yet) callable.

Panel A: Bond-level

Variables	N	Mean	SD	P05	P25	Median	P75	P95
Bond characteristics:								
Coupon rate (%)	5,493	8.0	2.3	4.5	6.2	8.0	9.6	11.8
Bond initial rating	5,493	B+	-	BB+	BB-	B+	B-	CCC
Time to maturity	5,493	8.8	3.4	5.0	7.1	8.1	10.0	11.0
Issuance amount ($M)	5,493	447	421	75	200	325	550	1,250
Issuer characteristics:								
Debt-to-Asset Ratio (%)	4,865	51.9	28.0	19.1	34.9	47.5	62.3	101.3
Debt-to-EBITDA	4,611	4.8	57.7	-0.9	2.8	4.3	6.3	13.6
Total asset ($M)	5,124	9,029	21,823	344	1,379	3,413	8,912	29,610
# bonds by call type:								
Call type:	All	Fixed price		Make whole		Non callable		Hybrid
N	5,493	1,538		597		453		2,905

Panel B: Bond-month-level

Variables	N	Mean	SD	P05	P25	Median	P75	P95
Bond characteristics:								
Bond trading price	173,375	97.6	18.6	59.1	96.7	102.1	106.4	113.4
YTM (%)	173,375	8.6	6.7	3.0	5.3	6.9	8.8	24.7
Swap spread (%)	173,375	6.3	6.7	1.4	2.8	4.2	6.4	22.4
CDS (%)	106,134	4.4	5.5	0.4	1.3	2.6	4.9	17.8
Moneyness (%)	77,870	-11.4	14.7	-40.2	-18.2	-8.2	-1.4	7.3
Bond rating	306,640	BB+	-	AA	BB	BB+	B-	C
Year(s) to maturity	307,740	5.7	3.2	1.2	3.7	5.6	7.3	9.8
Outstanding ($M)	307,740	419	432	5	150	300	500	1,250
# observations by call status in month t:								
Call status:	All	Fixed price		Make whole		Not callable		
N	307,740	56,438		164,939		86,363		

- ”Make-whole callable” bonds can be called at a price equal to the present value of remaining contractual bond payments, discounted at the relevant US Treasury rate plus a “make-whole spread,” typically 5bps-25bps (resp. 50bps) for IG (resp. HY) issuers, subject to a floor at par (see Appendix Figure A-1). That is, make-whole calls are effectively prepayments with a penalty that almost entirely neutralizes the interest rate and credit spread optionality embedded in these bonds.

- ”Fixed-price callable” bonds typically feature a non-call period, following which they can be called at a fixed price specified in the relevant bond indenture. The call price schedule usually decreases over the bond’s life and converges to par at maturity; the level of the call price usually depends on the initial yield at which the bond is issued, as documented by Powers (2021).

- ”Hybrid” bonds are almost identical to fixed-price callable bonds, except for the fact that they allow the issuer, during the initial phase of its contractual life, to exercise a call at a make-whole price equal to the present value (discounted at the relevant US Treasury rate plus the make-whole spread) of contractual bond payments *up to the first fixed-price call date*. Thus, even during their make-whole call period, hybrid bonds embed an interest rate and credit spread option for the issuer.

Figure 1 shows, for the different call protection provisions defined above, the median call price for our HY bond sample as a function of (normalized) remaining time to maturity.[3] Call prices are highest for pure make-whole callable bonds and converge to par at maturity. For hybrid bonds, the call price is declining steeply during the make-whole call period and converges to the first fixed-price call price (typically at 50% remaining maturity, see discussion below). The call price schedule is a step function for fixed-price callable debt.

Callable bonds may include a “non-call period” during which the option cannot be exercised. Appendix Figure A-2 shows the length of this non-call period by bond type

[3]For simplicity of illustration, option price for bonds which are either fixed-price callable during the second half of bonds’ contractual life or make-whole callable during the second half of bonds’ contractual life are omitted. Those observation only count for a small fraction in our sample, as shown in Figure A-2 Panel A.

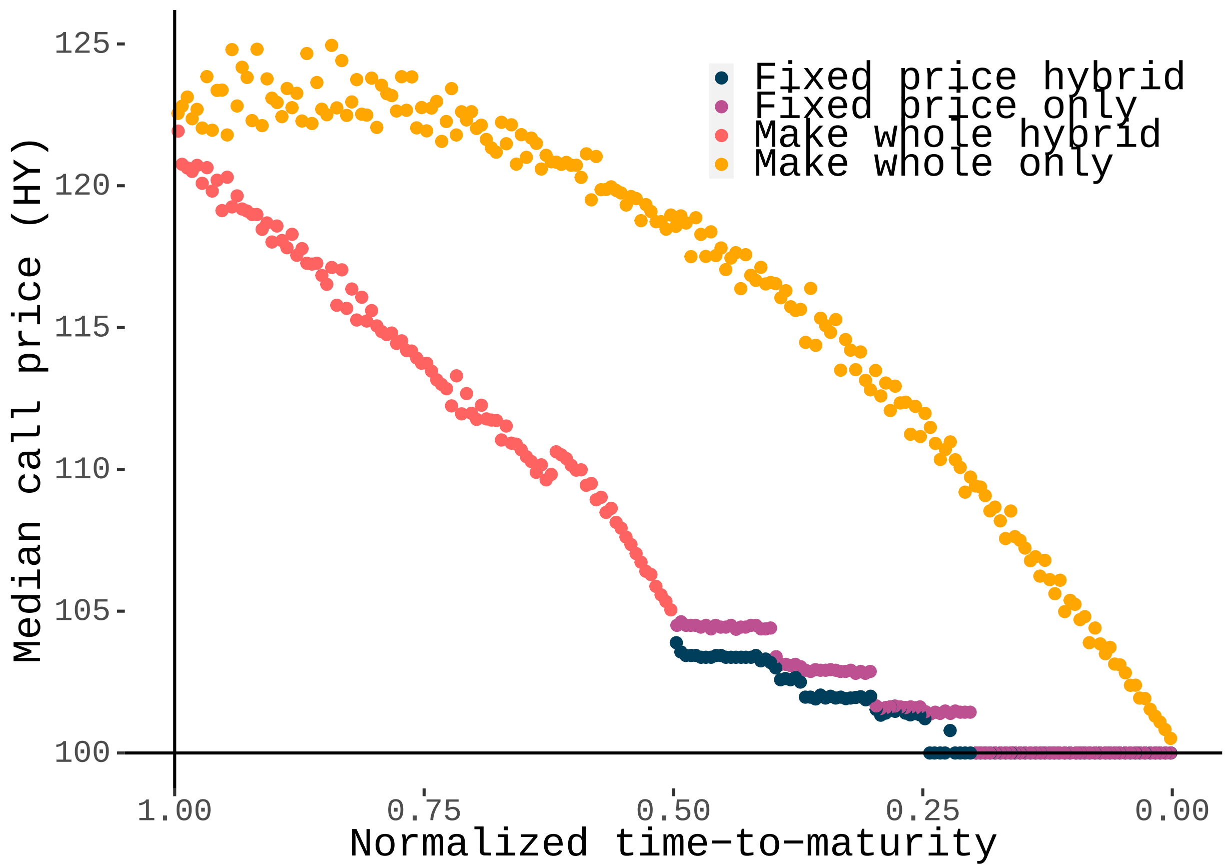


Figure 1: Median call price by option type
The figure displays the median call price as a function of remaining time to maturity bucket (normalized from 1.0 to 0.0 by initial maturity and in 0.001 decrements) for all callable bonds in our HY bond sample, distinguishing by call protection type.

for our sample of HY bonds (Panel A) and IG bonds (Panel B). Make-whole callable bonds rarely have a non-call period; since their call premium almost entirely neutralizes the embedded interest rate and credit spread options, issuers do not pay a significant yield premium for this type of optionality, and are thus not hesitant to issue debt that is callable immediately.[4] HY fixed-price callable bonds usually become callable half-way through their contractual life. Instead, IG fixed-price callable bonds become callable

[4]See Mann and Powers (2003) for a study of the premium paid by issuers for make-whole callable debt; for a small and early sample of make-whole callable bonds, the article estimates that issuers pay an extra 11 bps p.a. for the option.

somewhat earlier in their life; these bonds are however relatively uncommon in the IG market (see Figure 2). Finally, the make-whole call period of hybrid bonds usually starts as soon as the bond is issued, and for the HY sample those bonds become fixed-price callable typically half-way through their contractual life. IG hybrid bonds, in contrast, typically become fixed-price callable only shortly before maturity. For these latter bonds, the short fixed-price call period is effectively a “clean-up” call option for the issuer, who retains the flexibility to refinance outstanding debt at no cost typically within a year of its contractual maturity. Thus, while hybrid HY bonds have a non-negligible fixed-price call period (and thus embed a non-trivial interest rate and credit spread option), hybrid IG bonds are de-facto almost pure make-whole callable bonds and thus benefit from strong prepayment protection.

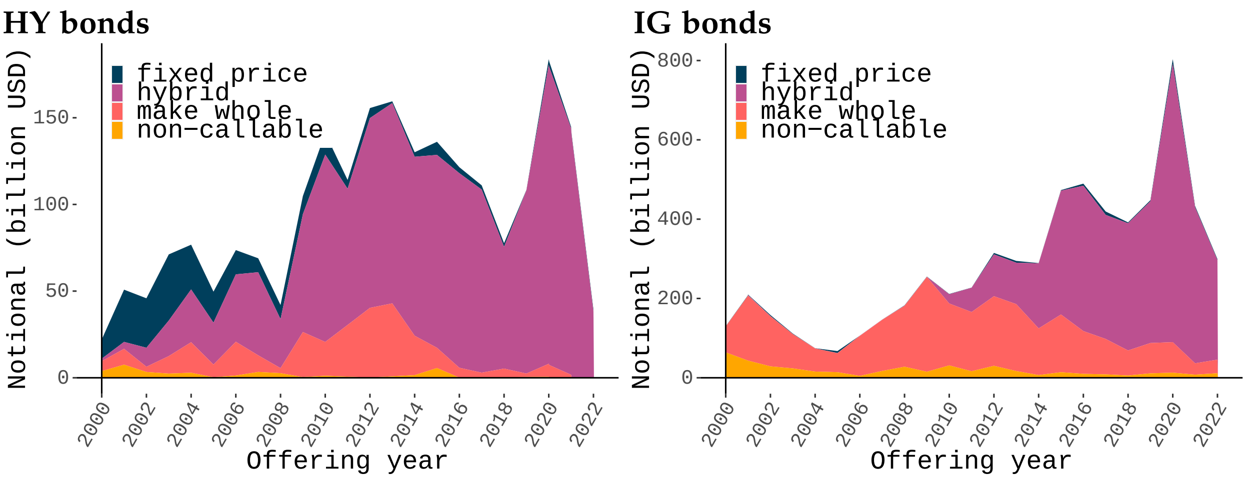


Figure 2: Bond call features over time
This figure plots the annual aggregate volume of newly issued HY bonds (left) and IG bonds (right) over time. The figure distinguishes between “fixed price” callable bonds, “make-whole” callable bonds, “non-callable” bonds, and “hybrid” bonds.

Figure 2 (left) shows the distribution of call protection structures for newly issued HY bonds in our sample over time. While close to 50% (by notional amount) of newly issued bonds had a fixed-price callable structure before the GFC (with the balance consisting of mostly hybrid and make-whole callable bonds), the vast majority of HY bond issuances in the sample post GFC consists of hybrid bonds.

Figure 2 (right) shows the same figure for IG bonds, for comparison. Close to 100%

of newly issued bonds were non-callable or pure make-whole callable before the GFC. But a trend similar to that in the HY market has emerged, with hybrid bonds (with strong prepayment protections, as discussed previously) nowadays being the dominant contract for IG issuers.[5]

To summarize, the IG and HY markets differ in an important dimension: while most IG bonds benefit from strong call protections, by contrast most HY bonds, given their hybrid structure, embed an interest rate and credit spread option. This has two important implications. First, empirical studies of the determinants of corporate bond spreads (for e.g. Collin-Dufresn, Goldstein and Martin (2001) or He, Khorrami and Song (2022)) typically exclude callable bonds with weak prepayment protections, so as to properly measure the yield compensation that investors get paid for being exposed to credit risk. Since almost all HY bonds are callable — with a non-trivial amount of embedded interest rate and credit spread optionality — these studies exclude the entire HY market from their analysis. Second, several articles (for e.g. Becker et al. (2022)) have attempted to retrieve, using reduced form regressions and the difference in initial yield between callable and non-callable debt issued by the same firm around the same time, a measure of compensation paid to investors for being short such option. Our findings highlight the difficulty of this exercise nowadays, given that HY firms only issue callable debt.

We next argue that HY firms not only exercise these options to reduce their debt interest expense, but also as a device to manage the maturity structure of their debt and mitigate rollover risk. Since the IG bond market is perceived to be less fragile than its HY counterpart, IG firms have no need to pay for the optionality provided by callable debt, and thus restrict themselves to debt with strong prepayment protection for investors.

5.2 Prepayments in credit markets

We turn to the main objective of our study—HY firms' bond prepayment behavior. In our sample, HY bonds have an unconditional annual prepayment probability of 12.1%; given the average (median) bond contractual maturity of 8.8 years (resp. 8.1 years), the vast majority of HY bonds are retired before their contractual maturity. Figure 3 shows the

[5]Internet Appendix Figure IA-1 shows the figure on an equal-weighted basis. The distribution of call protection structures over time by number of bonds is very similar to the distribution by notional amount.

time-series of bond prepayment rates. Prepayment rates are negatively correlated with HY credit spread (correlation of -64%) and with the 10-year US treasury rate (correlation of -29%), drop sharply during the GFC, and are significantly greater than those in the IG bond market (around 3.3% p.a.).

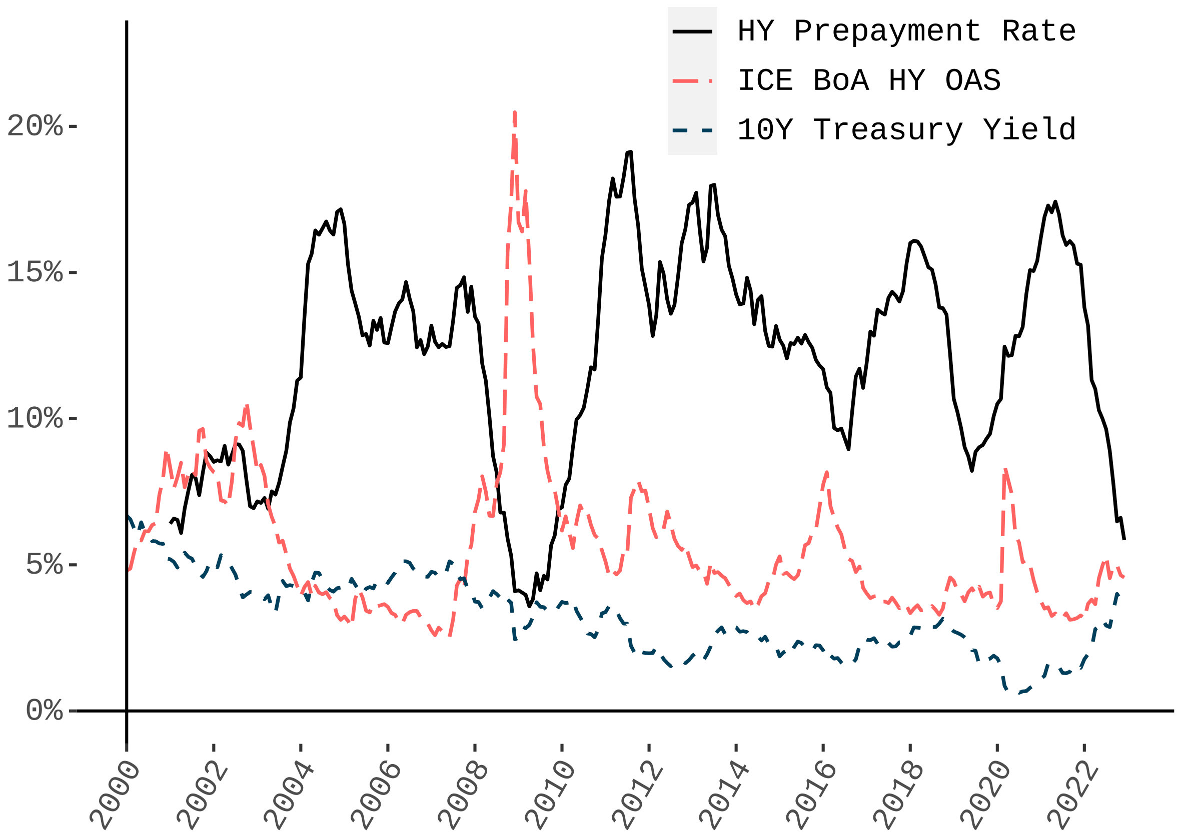


Figure 3: Prepayment rates in the HY bond market
The figure displays annual prepayment rates in the market for HY bonds from 2000 to 2022. Prepayment rates are defined as the total (market-wide) notional amount that is prepaid in the respective trailing 12 months period scaled by the total amount outstanding in the previous year. The red long-dashed line indicates ICE BoA HY OAS, and the blue short-dashed line indicates market yield of US 10-Year Treasury.

5.2.1 Survival Curves

We next compute and plot in Figure 4 survival curves conditional on the call protection type of the bond for our HY sample. Fixed-price callable and hybrid bonds behave similarly—about 37% of these bonds are prepaid on or before half their contractual life, and 81% of them are prepaid with 25% of the bond's original life remaining. The slope of the survival curves steepens significantly around half-way through the contractual life. This steepening coincides with (i) the end of the non-call period (for fixed-price callable bonds) and (ii) the switch from make-whole to fixed-price callability (for hybrid bonds), after which the call exercise becomes cheaper from the issuer standpoint.

Instead, non-callable bonds and pure make-whole callable bonds are prepaid much less frequently: only 37% and 23% respectively of these bonds prepaid on or 25% of the bond's original life remaining, and around 50% of notional is left outstanding at contractual maturity.

One could wonder why our survival curves are downward sloping for bonds benefiting from strong prepayment protection — for instance fixed-price callable bonds during the first half of their contractual life (during which the bond is usually not callable), non-callable bonds (which, by definition, cannot be called, except under special circumstances) or makewhole callable bonds (featuring steep call premia for the issuer). In Table 2 we shed light on this question, by providing information on the prepayment methods: (i) call option exercises, (ii) tender offers, or (iii) open market repurchases. For all these bond types for which a call exercise is very (and sometimes infinitely) costly, the decline in survival curve is almost entirely attributable to tender offers and open market repurchases.[6]

That being said, for about 17% of hybrid bonds, the issuer does exercise the call option during the make-whole call period. Similarly 33% of make-whole callable bonds do experience at least one call event (usually occurring in the last quarter of the bond's life, i.e., 2-3 years before maturity for a typical 10-year bond).[7] This suggests that firms might

[6]In particular, the decline in survival curve during periods of strict prepayment protection is not due to defaults; while prepayment rates in our HY sample average 12.1% p.a., average default rates are instead below 2% p.a.

[7]Figure IA-3 in the Internet Appendix shows the total bond volume that is prepaid per year by action type. Before the GFC almost all prepayments (by volume) are either tender offers or fixed-price calls. After the GFC, an increasing number of make-whole callable and hybrid bonds are called (during their make-

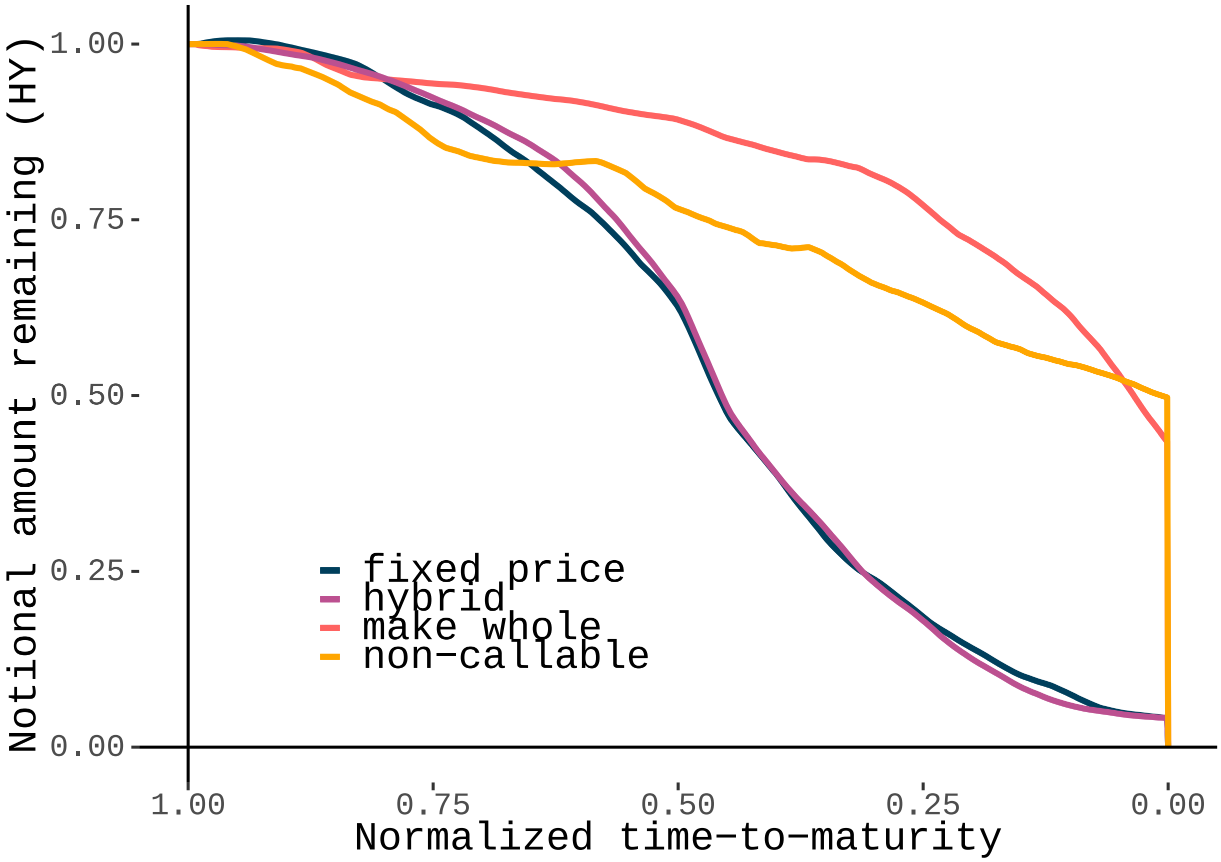


Figure 4: Survival curves by call protection type
The figure displays the average percentage of offering amount remaining outstanding as HY bonds progress towards scheduled maturity. The figure distinguishes between "fixed price" callable bonds, “make-whole” callable bonds, "non-callable" bonds, and "hybrid" callable bonds. The x-axis is normalized by the contractual bond time to maturity at issuance.

sometimes exercise their call options for reasons other than interest rate considerations—something we will explore in more detail in the remainder of this paper.

whole period for hybrid bonds). Table IA-1 in the Internet Appendix reports statistics about prepayment size (in % of initial offering amount) by bond type and prepayment method. When call options are exercised typically a sizable share of the initial offering amount is prepaid, ranging from 68% for fixed-price callable bonds to 77% for hybrid bonds. Similarly, tender offers are large, with the median values ranging from 36% for non-callable bonds to 95% for fixed-price callable bonds. In contrast, open market repurchases tend to be small, with about 24% to 43% of the notional being retired per buyback event.

Table 2: Prepayment events by call protection type

This table shows statistics about prepayment events by bond type. The sample comprises all high-yield bonds that are outstanding at any point during the 2000 to 2022 period. *Any* indicates the fraction of bonds that have at least one prepayment event, irrespective of prepayment type. *Call*/*Tender*/*Buyback* indicates the fraction of bonds that have at least one call option exercise/tender offer/open market buyback. # *Events | > 0* is the average number of prepayments (irrespective of type) for bonds that have at least one prepayment event. For callable bonds, we further distinguish between events before and during the call period. For hybrid bonds, we additionally distinguish between events during the make-whole period and the fixed-price period. Note that *Call + Tender + Buyback* does not add up to *Any* as some bonds have multiple events of different type. Further note that *call period: no + call period: yes* does not add up to the overall (combined) period as some bonds have events both before and during the period in which the bond is callable.

		Prepayment Event (0/1)				
	# Bonds	Any	Call	Tender	Buyback	# Events | > 0
Non-callable	453	21.0%	3.1%	17.2%	4.4%	95
Fixed price	1,538	73.6%	64.2%	31.1%	9.9%	1,132
call period: no		29.0%	5.1%	20.9%	7.8%	
call period: yes		76.1%	73.9%	15.3%	3.6%	
Hybrid	2,905	62.4%	55.8%	22.9%	7.7%	1,812
call period: no		4.9%	4.0%	1.6%	0.9%	
call period: yes (MW)		27.5%	16.8%	12.0%	6.0%	
call period: yes (fixed)		69.1%	66.3%	18.1%	3.5%	
Make whole	597	49.6%	33.2%	25.0%	10.6%	296
call period: no		0.3%	0.3%	0.0%	0.0%	
call period: yes		49.6%	33.1%	25.0%	10.6%	

5.2.2 Call option moneyness and the interest cost savings channel

HY issuers driven by the interest cost savings motive are expected to exercise the call option embedded in their bond more frequently when such option is sufficiently "in the money". While the determination of the optimal call strategy is complex and requires a structural model (which we will eventually develop in Section 6), we first provide a simplified (and model-free) empirical analysis that allows us (a) to compute a measure of "moneyness" of such option, and (b) to study firms' propensity to prepay as a function of moneyness. As in the equity derivatives' literature, moneyness in our framework captures the intrinsic value of the call option, if such option was expiring today. We compute moneyness for bond i at time t as the relative difference between (i) the option strike price at such time and (ii) the synthetic non-callable value of the bond, i.e.,

$$\text{Moneyness}_{i,t} = \left(P_{i,t}^{\textit{non-call}} - P_{i,t}^{\textit{call}}\right) / P_{i,t}^{\textit{call}}.$$

$P_{i,t}^{call}$ is the price at which bond i can contractually be called in period t — in other words, the *strike price* of the option. $P_{i,t}^{non\text{-}call}$ is the synthetic non-callable value of the bond — a measure of the *underlying spot price*.[8] We measure $P_{i,t}^{non\text{-}call}$ by discounting the bond's contractual future cashflows (to maturity) at the yield at which firm i could issue non-callable debt with identical time to maturity. Since most HY firms only issue callable debt, we do not observe such counterfactual non-callable bond yield; instead, we construct such yield by using the price of non-callable credit instruments — single-name CDS — adjusted for the bond-CDS basis. In other words:

$$P_{i,t}^{\textit{non-call}} = \sum_{n=1}^{N} \frac{CF_{i,t+n}}{(1 + r_{i,t}^{*})^{n}},$$

where $CF_{i,t+n}$ is bond i's contractual future cash-flow at time $t+n$, discounted at

$$r_{i,t}^{*} = \textit{swap rate}_{i,t} + \textit{CDS spread}_{i,t} - \textit{CDS bond basis}_{t}.$$

$\textit{swap rate}_{i,t}$ is the x-year (interpolated) swap rate in month t, with x the remaining time to maturity for bond i in period t. $\textit{CDS spread}_{i,t}$ is the x-year (interpolated) CDS spread in month t for the *issuer* of bond i. $\textit{CDS bond basis}_{t}$ is the three month moving average of median difference between the *CDS spread* for HY issuers in our sample and the corresponding bond *swap spread* (i.e., yield-to-maturity minus swap rate).[9] Our moneyness

[8]The synthetic non-callable value is the hypothetical market value of the bond is such instrument was not callable. This is the relevant object for the purpose of computing the option intrinsic value, since firms, when making call exercise decisions driven by interest cost savings incentives, are implicitly choosing the market price-minimizing strategy: either (i) keep the bond outstanding — with a value equal to the hypothetical market value of that bond — or (ii) retire the bond at the relevant call price. Importantly, the spot price is *not* the current trading price, which is an equilibrium object taking into account the market's perceived call probability of the instrument.

[9]We winsorize our interpolated *CDS spread* and *swap spread* results at 1% level on both sides to avoid extreme values. Only bonds benefiting from strong prepayment protections — i.e. non-callable and pure make-whole callable bonds — are used in the CDS-bond basis calculation. Figure A-3 in the

calculation requires that CDS price data is available for the bond issuer, as well as the bond issue to be senior unsecured.[10] Appendix Table A-2 compares bonds issued by firms with versus without CDS data. We have CDS data for about 50% of the firms in our sample. While firms with CDS data are larger, they are not substantially different w.r.t. to credit rating and coupon rates. Bonds issued by firms with CDS data are more likely to include hybrid call provisions and less likely to include fixed-price call provisions. This is due to our CDS coverage, which increases in the later part of the sample when hybrid call provisions become more popular, (see Figure 2).

Figure 5 shows the moneyness distribution (at the instrument-month level) for our HY bond sample. We notice a striking discontinuity near zero moneyness ("at-the-money"), i.e., we see only few bonds that remain outstanding if they are callable and the option is in-the-money. As we will see next, HY firms exercise the call option embedded in their bonds aggressively when such option becomes in-the-money, leading to this pattern of the moneyness distribution.

Appendix shows our computation of the CDS-bond basis, separately for IG and HY credit markets over time. Consistent with what has been documented in the literature (see, e.g., Bai and Collin-Dufresne, 2019), the CDS-bond basis is around zero before the GFC, drops sharply during the GFC, and remains negative afterwards. Moreover, the HY CDS-bond basis is consistently wider (i.e. more negative) than its IG counterpart.

[10] We remove 664 HY senior secured bonds from the sample for this analysis as CDS generally reference senior unsecured debt.

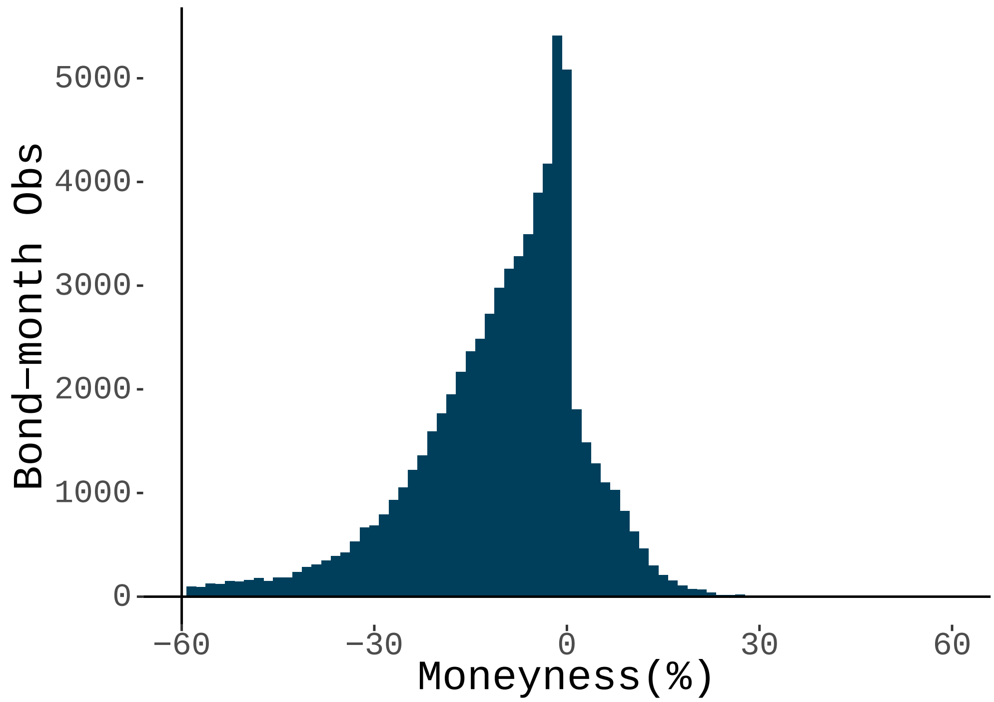


Figure 5: HY call option moneyness
This figure plots the number of bond-month observations by option moneyness for callable HY bonds.

We then examine the relationship between call option exercises and option moneyness by estimating the following model:

$$Prepay^{call}_{i,j,t} \sim \sum_{m=1}^{M} \beta_m * \mathbb{1}^{m}_{i,j,t} + \alpha_j + \alpha_t + \epsilon_{i,j,t} \quad (1)$$

where $\mathbb{1}^{m}_{i,j,t}$ is an indicator equal to one if the call option moneyness for bond i (issued by firm j) in month t belongs to moneyness bin $m \in \{1, ..., M\}$, and where we bin moneyness in intervals of 2.5% (or 5% when moneyness is greater than or equal to 0%). α_j and α_t are issuer and year fixed effects, respectively. $Prepay^{call}_{i,j,t}$ indicates if bond i has been called by issuer j in month t.

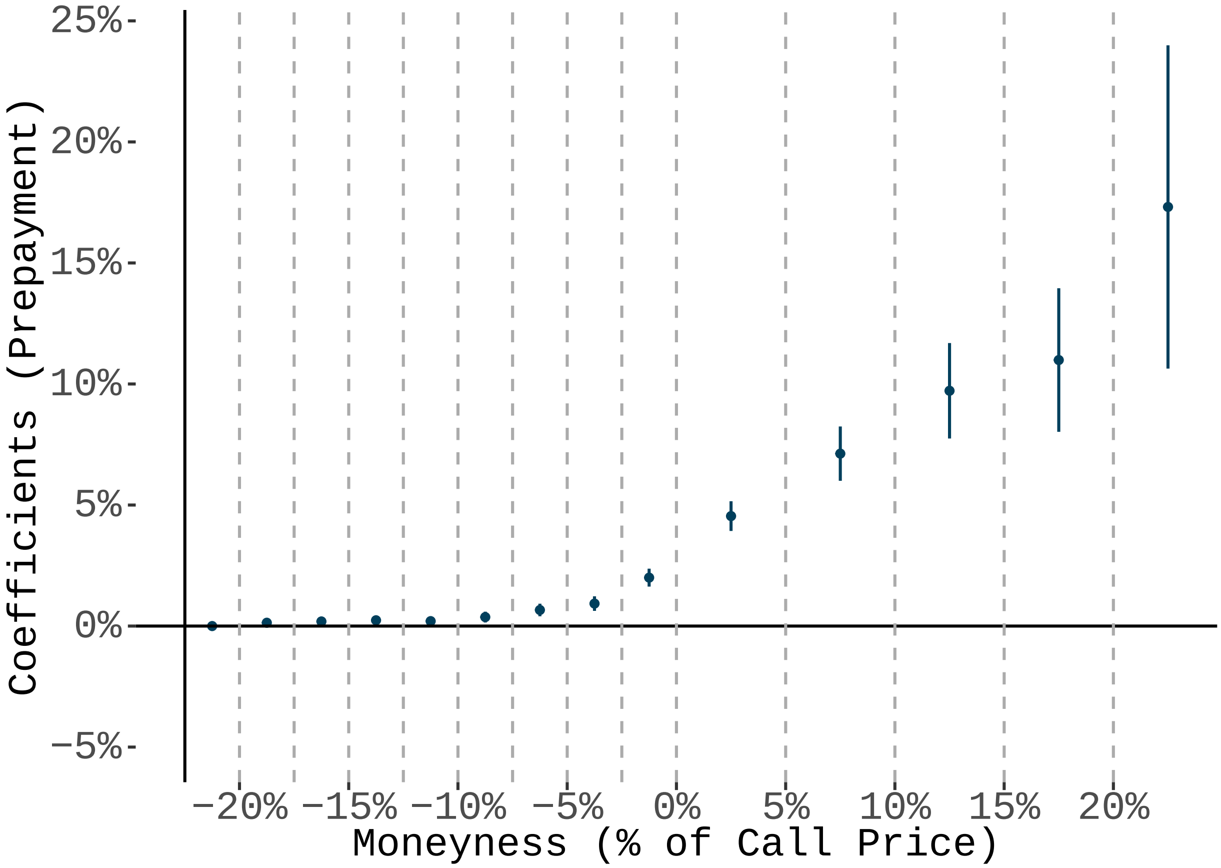


Figure 6: Option moneyness and call option exercise probability
This figure shows regression coefficients of an estimation of bond prepayment probabilities by option moneyness dummy, where each dummy indicates the option moneyness is within the boundaries represented by the adjacent vertical dashed lines. The reference category is moneyness < 20% on the left. The sample includes HY bonds only. The figure shows 95% confidence intervals around the coefficient estimates.

We plot in Figure 6 the estimated coefficients for the moneyness bins. There is a sharp increase in the probability of a bond being prepaid in a given month around the at-the-money threshold, indicating that firms exercise the interest rate and credit spread options embedded in their callable debt aggressively, once such option has intrinsic value. The economic magnitudes are large. Our linear probability model suggests that a switch from out-of-the-money to in-the-money increases the monthly prepayment probability by 7.5-10 p.p. (or 61-72 p.p. annualized).[11] Of course one should not necessarily expect a bond whose moneyness is positive to be immediately called: the option embedded in callable bonds has an intrinsic value (measured by moneyness in our case) as well as time value, and a firm whose callable debt is in-the-money might still find it optimal to wait before exercising it. In fact, the greater the option time value, the greater the moneyness should need to be in order for the firm to find it optimal to exercise its call. We test this hypothesis in appendix Figure IA-2, by including in model (1) a control for the time-to-maturity of the bond (a proxy for the time value of the option), as well as the interaction between this time-to-maturity control and moneyness. We verify that bonds with greater remaining time-to-maturity tend to be prepaid at greater levels of moneyness, relative to bonds with lower remaining time-to-maturity —- consistent with the idea that the option embedded in callable debt has time value and that firms exercise these options at greater levels of intrinsic value when the time value is higher.

Overall, our results indicate that interest rate and spread motives — and thus the interest cost savings channel — are strong drivers of firms' prepayment decisions. For comparison, Berger et al. (2021) estimate a similar model for refinancing decisions made by US households that have fixed-rate prepayable mortgages. In contrast to firms, household prepayment behavior is sluggish, i.e., households only start refinancing once potential interest cost savings are non-trivial and even then, the estimated increase in prepayment hazard is only 2% per month. This suggests that prepayment frictions — whether behavioral, or financial — are substantially larger for households than for firms.

5.2.3 The Rollover Risk Management Motive

In this section, we show evidence that a second important channel is at play when explaining bond prepayment behavior: the rollover risk management motive. To do this, we use a two-pronged approach, by focusing on call option exercise probability and (1) how it relates to measures of rollover risk controlling for the interest cost savings incentive, and (2) how, for bonds with negative moneyness, this probability relates to rollover risk.[12]

[11] A 10% prepayment probability per month leads to a $1-(1-0.1)^{12}=72\%$ prepayment probability.

[12] In Figure IA-11 of the Internet Appendix, we confirm that most call option exercise are associated with a refinancing — rather than a deleveraging — by performing event studies around call dates, and by

Our first strategy relies on estimating the following model:

$$Prepay^{call}_{i,j,t} \sim \beta_0 Rollover_{j,t-1} + \sum_{m=1}^{M} \beta_{1,m} \mathbb{1}^{m}_{i,j,t} + \alpha_j + \alpha_t + \epsilon_{i,t}, \tag{2}$$

where $Rollover_{j,t-1}$ is a measure of rollover risk for firm j at time $t-1$. We expect a positive β_0 as firms facing high rollover risk should be more likely to refinance early to avoid that markets are closed at maturity. We control for moneyness bins to hold fixed the strength of the interest cost savings incentives and isolate the rollover risk motive. Issuers with a higher proportion of debt that is due soon face higher rollover risk (see, e.g., Gopalan, Song and Yerramilli, 2014; Valenzuela, 2016; Almeida et al., 2012). Motivated by this observation, we define:

$$Rollover_{j,t} = \frac{dd^1_{j,t} + dd^2_{j,t}}{dd^1_{j,t} + dltt_{j,t}},$$

where dd^1 is the long-term debt that matures within one year; dd^2 is the long-term debt that matures in more than one year, but within two years; and $dltt$ represents total long-term debt that matures in more than one year. All information is directly available from Compustat. A similar ratio can be calculated using the monthly bond outstanding balances we constructed based on information from Mergent FISD. For ease of interpretation we use indicator variables that are equal to one if $Rollover_{j,t}$ is $> 0\%$, $> 10\%$, or $> 20\%$, and zero otherwise, when estimating model (2).

Table 3, Panel A, shows that firms with higher rollover risk are more likely to call their bond early, after controlling for option moneyness. The effect is economically large. The coefficient estimates from columns 3 and 6 imply that firms that have more than 20% of their long-term debt due within in the next two years have a 0.4 p.p. (or 4.7 p.p. annualized) higher monthly probability to call their debt early *after controlling for the strength of the interest cost savings channel*.

In our second approach, we examine the effects of rollover risk on the likelihood that

showing that (a) debt-to-asset ratios post-call exercise only decline by 1 p.p. on average, relative to their level three months prior to such exercise (for a mean debt-to-asset ratio of 51.5%), and (b) the distribution of quarterly debt-to-asset ratio changes around call events, while having slightly greater kurtosis, is otherwise statistically close to that outside such call events.

Table 3: Rollover risk and call option exercise probability

Panel A of this table shows estimated coefficients from model (2). The dependent variable is an indicator variable that is equal to one if bond i issued by firm j is called in period t, and zero otherwise. *Rollover* is the fraction of long-term debt by firm j that is due within the next two years. In columns 1 to 3, *Rollover* is estimated based on Compustat data; in columns 4 to 6, *Rollover* is estimated based on monthly bond outstanding balances from Mergent FISD. We use indicator variables ("*High rollover risk*") that are equal to one if *Rollover* is $> 0\%$, $> 10\%$, or $> 20\%$, and zero otherwise. Panel B of this table shows estimated coefficients from model (3). *Moneyness* $<= -2.5\%$ is an indicator variable that is equal to one if the call option embedded in bond i has a moneyness of $<= -2.5\%$ in period t, and zero otherwise. *Interaction* is the interaction term between the "out-of-the-money indicator" (*Moneyness* $<= -2.5\%$) and the indicator variable for high rollover risk, as described above. Robust standard errors clustered at the firm level are in parenthesis.

Panel A.

Dependent Var.:	Prepaid by call$_{i,j,t}$ (0/1)					
	Compustat			Mergent FISD		
Rollover$_{j,t-1}$:	>0%	>10%	>20%	>0%	>10%	>20%
	(1)	(2)	(3)	(4)	(5)	(6)
High rollover risk	0.016***	0.004***	0.004***	0.007***	0.003*	0.003
	(0.005)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Moneyness Bins	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	58,954	58,954	58,954	58,954	58,954	58,954
R^2	0.074	0.074	0.074	0.079	0.079	0.079

Panel B.

Dependent Var.:	Prepaid by call$_{i,j,t}$ (0/1)					
	Compustat			Mergent FISD		
Rollover$_{j,t-1}$:	>0%	>10%	>20%	>0%	>10%	>20%
	(1)	(2)	(3)	(4)	(5)	(6)
Interaction	0.027**	0.008**	0.007	0.013***	0.016***	0.015***
	(0.013)	(0.004)	(0.005)	(0.004)	(0.004)	(0.004)
Moneyness $<= -2.5\%$	-0.042^{***}	-0.042^{***}	-0.040^{***}	-0.046^{***}	-0.044^{***}	-0.041^{***}
	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)
High rollover risk	-0.008	-0.004	-0.002	-0.006	-0.010^{**}	-0.009^{*}
	(0.013)	(0.004)	(0.005)	(0.004)	(0.004)	(0.005)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	58,954	58,954	58,954	58,954	58,954	58,954
R^2	0.053	0.053	0.053	0.060	0.060	0.060

a firm calls a bond at a time when interest cost savings incentives are weak. In that case, firms would not be lowering their cost of debt by exercising their call options. If rollover risk is high, however, firms might have an incentive to retire and refinance their debt early even if the embedded option is out-of-the-money. To test this conjecture we estimate the following model:

$$\begin{aligned} Prepay^{call}_{i,j,t} \sim & \beta_0 * Interaction_{i,j,t-1} + \beta_1 * Rollover_{j,t-1} \\ & + \beta_2 * (Moneyness \leq -2.5\%)_{i,j,t} + \alpha_j + \alpha_t + \epsilon_{i,t}, \end{aligned} \tag{3}$$

where $(Moneyness \leq -2.5\%)$ is an indicator variable that is equal to one if the call option embedded in bond i has a moneyness below -2.5% in period t, and zero otherwise. *Interaction* is the interaction term between the "out-of-the-money indicator" $(Moneyness \leq -2.5\%)$ and *Rollover*, as defined above. β_0, the coefficient of interest, indicates whether the likelihood that a firm calls an out-of-the-money option varies with the degree of rollover risk that the issuer faces.

Our model estimation, reported in Table 3, Panel B, suggests that firms are significantly less likely to exercise call options that are out-of-the-money. This effect, however, is weaker in the presence of greater issuer-level rollover risk. The estimates reported in columns 2 and 6, for instance, indicate that out-of-the-money options are around 0.8-1.6 p.p. (10.0-20.9 p.p. annualized) more likely to be called when the issuer has more than 10% of their long-term debt due within the next two years.

6 Theory

Given our empirical evidence, in this section we build a theory of firms' debt refinancing decisions. Its objective is to rationalize the observed firms' early refinancing behavior in HY credit markets, and to help disentangle the various motives that drive a firm's management to prepay debt early and pay out costly call premia to debt investors. We narrow down our focus onto the two main drivers of refinancing decisions we have been studying until now: the *interest cost savings* motive, and the *rollover risk management* motive. An estimated version of our model will then allow us to compute counterfactuals that are unavailable when relying solely on reduced form empirics.

We begin with a stripped down version of our model, solely focused on rollover risk, and meant to illustrate the trade-off faced by firms in the presence of credit supply shocks. In that model, a firm fears that it might not be able to refinance its debt when it comes due, possibly due to the occurrence of a sudden stop in debt capital markets. In order to mitigate this risk, the firm issues callable debt and chooses an optimal time at which to pay a call premium and early refinance its outstanding debt. Interest rates and credit spreads are kept constant, in order to facilitate our exposition of this *rollover risk management* channel. We then re-introduce stochastic interest rates and credit spreads in Section 6.2, so that callable debt is now valuable not only as a way to mitigate market shut-down risks, but also since it embeds a call option on interest rates and the firm's credit spread. Our model is meant to analyze the trade-offs and timing of callable debt prepayment decisions; we take the contractual structure of the bonds issued as given, and leave the study of the optimal debt contract for future research.

6.1 Rollover risk in a simple setting

6.1.1 Setup

Time t is continuous. The environment is characterized by good (state 1) or bad (state 0) credit market conditions, switching from one state to another according to transition intensities θ_u and θ_d respectively. Denote s_t this discrete state continuous time Markov chain.[13] A firm receives cashflows y per unit of time; it rolls over bullet-maturity debt with notional amount b, coupon rate c (with $c > r$, the risk-free rate) and maturity m whenever credit market conditions are good; instead, financial markets are shut down in bad credit market environments and a firm whose debt comes due at such time is forced to default. At time t, we denote $T_t \in [0, m]$ the time-to-maturity of the firm's outstanding debt. Finally, we assume that the firm, outside the possibility of default at the maturity of its debt, also defaults at Poisson arrival rate λ; upon default, creditors recover a fraction

[13] s_t could be an idiosyncratic shock encoding a firm's fundamentals; a deterioration of these fundamentals could lead to the violation of debt covenants and the inability for the firm to access credit. Alternatively, s_t could be an aggregate shock that triggers a sudden stop in credit markets. The precise interpretation of this shock does not impact any of our conclusions, since our model does not feature any general equilibrium feedback from firms' aggregate actions into prices. That being said, given the data on aggregate HY debt issuances (see Figure IA-12), our preferred interpretation is that s_t is an aggregate state variable affecting all firms at the same time.

$\alpha \in (0,1)$ of the underlying project value, while shareholders are wiped out. While the parameters y, r, b, λ, α and m are exogenous, the bond coupon c will be determined endogenously so as to insure that the firm's debt is always issued at par.

Firm's shareholders choose a debt refinancing strategy; they balance the benefits of early refinancing (which reduces the likelihood of the firm's future default) vs. its cost — the call premium $\kappa(T)$ (per unit of outstanding debt) that the firm must pay when refinancing T years before the bond's contractual maturity.[14] Our results are general to any type of call premium function, so long as the following assumptions are satisfied.

Assumption 1 *The call premium $\kappa(T)$, as a function of time-to-maturity T, is such that (i) $\kappa(T) \geq 0$, (ii) $\kappa(T)$ is increasing and differentiable in T, and (iii) $\kappa(0) = 0$.*

Given aggregate state s and time-to-maturity T, denote $E_s(T)$ (resp. $D_s(T)$) the equity value (resp. debt price per unit of face value). The relevant state space is $(s, T) \in \{0,1\} \times [0, m]$. Firm's management (acting on behalf of shareholders) solves

$$E_s(T) := \sup_{\boldsymbol{\tau}} \mathbb{E}_{s,T} \left[\int_0^{\tau_d} e^{-rt} (y - cb) \, dt - b \sum_{j \geq 0, \tau_j \leq \tau_d} e^{-r\tau_j} \kappa(T_{\tau_j}) \right], \tag{4}$$

where the subscript on the expectation indicates it is conditional on the current state being (s, T), where $\boldsymbol{\tau} = \{\tau_j\}_{j \geq 0}$ is a sequence of (calendar) refinancing times, and τ_d is the firm's (calendar) default time — such default can either be due to (i) a jump to default (at Poisson arrival rate λ), or (ii) a default at a debt maturity date, if the firm needs to rollover at a time when capital markets are closed. The equity market value is equal to the expected net present value of future net income $y - cb$, minus prepayment penalties $\kappa(T_{\tau_j})$ paid to creditors. Our formulation of the equity value implicitly assumes

[14] For example, if the firm issues make-whole callable debt, $\kappa(T)$ corresponds to the make-whole amount, computed at the risk-free interest rate plus a spread ς (typically 50bps for high-yield firms, as documented in Appendix Figure A-1). In that case, the premium $\kappa(T)$ satisfies

$$\kappa(T) = \int_0^T e^{-(r+\varsigma)t} c dt + e^{-(r+\varsigma)T} - 1 = \left(\frac{c}{r+\varsigma} - 1 \right) \left(1 - e^{-(r+\varsigma)T} \right).$$

If instead the firm issues fixed-price callable debt, then $\kappa(T)$ is the price premium over par, specified in the contractual document, at which the firm is allowed to call its outstanding debt at a given point in time. During the non-call period, one can then simply set $\kappa(T) = +\infty$.

that whenever the firm rolls over its debt, it does so at a price of par — meaning that such refinancing does not lead to either rollover gains or losses.[15]

Debt investors take shareholders' prepayment strategy $\boldsymbol{\tau}$ as given. If we denote τ_r the first refinancing time in the sequence $\boldsymbol{\tau}$, investors price callable bonds according to

$$D_s(T) := \mathbb{E}_{s,T}\left[\int_0^{\tau_r\wedge\tau_d\wedge T} e^{-rt}c\,dt + e^{-r(\tau_r\wedge\tau_d\wedge T)}\left(1_{\{T\leq\tau_r\wedge\tau_d\}}\right.\right.$$
$$\left.\left.+1_{\{\tau_r\leq T\wedge\tau_d\}}\left(1+\kappa(T-\tau_r)\right)+1_{\{\tau_d\leq\tau_r\wedge T\}}\frac{\alpha y}{rb}\right)\right], \quad (5)$$

where $\wedge$ indicates the minimum operator. The debt price is equal to the expected net present value of all coupon payments, until the earlier of (i) the maturity date, (ii) the refinancing date or (ii) the default time, whichever occurs first. If refinancing occurs first, creditors receive par plus the call premium $\kappa(T-\tau_r)$; if the bond matures first, creditors receive their principal back; otherwise if default occurs first, creditors receive the recovery value $\rho := \alpha y/(rb)$. In order to insure that the firm issues debt at a price of par, the coupon rate must satisfy the condition

$$D_1(m) = 1. \quad (6)$$

Given our focus on Markov strategies, the firm optimally uses a cutoff policy: it refinances as soon as capital market conditions permit and the time-to-maturity T falls below a critical value T^*. An equilibrium of our model is defined as follows.

Definition 1 *A credit markets' equilibrium (thereafter, an "Equilibrium") is (i) an optimal cutoff refinancing policy T^* and equity value function $E_s(T)$ that solve (4), (ii) a debt price function $D_s(T)$ that solves (5), and (iii) a bond coupon c so that (6) is satisfied.*

Under our Equilibrium definition, shareholders of the firm maximize equity value by finding the optimal refinancing policy, taking into account debt prices — in particular, by rolling over par-priced debt. Creditors price the firm's debt rationally, taking into

[15] This distinguishes our model from a literature that studies rollover losses — see for instance Leland (1998) or He and Milbradt (2014). This also makes our model "closer" to the data than most of the literature — since empirically, firms tend to issue debt priced at par with fixed coupons that are set so as to insure that the market clears at such par price at the time of issuance.

account firms' refinancing behavior; the coupon c is then set to satisfy condition (6): whenever capital markets are open, a firm that refinances issues debt at a price of par.

6.1.2 Model solution

In this simple environment, we can provide a sharp characterization of the Equilibrium. The following theorem delivers the key economics of the model.

Theorem 1 *In an Equilibrium, debt prices and equity values admit analytic expressions provided in Appendix A.3.1. The optimal refinancing time-to-maturity T^* satisfies*

$$\partial_T E_1(T^*) = -b\partial_T \kappa(T^*). \tag{7}$$

The optimal refinancing time solves the implicit equation (7): the left-hand side is the equity marginal value of time-to-maturity $\partial_T E_1$, whereas the right-hand side is the marginal prepayment cost $-\partial_T(b\kappa(T))$. Naturally, at the optimal prepayment time-to-maturity T^*, the firm equalizes marginal cost to marginal benefit.

Our model allows us to think about the extent to which callable debt enables the firm to hedge its exposure to market shutdown risk, and to quantify the value derived from the ability to issue state-contingent (rather than non-state-contingent) securities. In particular, we can define three different equilibria, depending on whether rollover risk is present or not, and on whether the firm is allowed to issue callable debt:

- the *callable debt equilibrium* will correspond to the environment in which firms face rollover risk and can issue callable debt;
- the *non-callable debt equilibrium* will correspond to the environment in which firms face rollover risk and can only issue non-callable, bullet-maturity debt;
- the *no-rollover risk equilibrium* will correspond to the environment in which firms do not face rollover risk (in which case the particular choice of debt is not important for enterprise value).

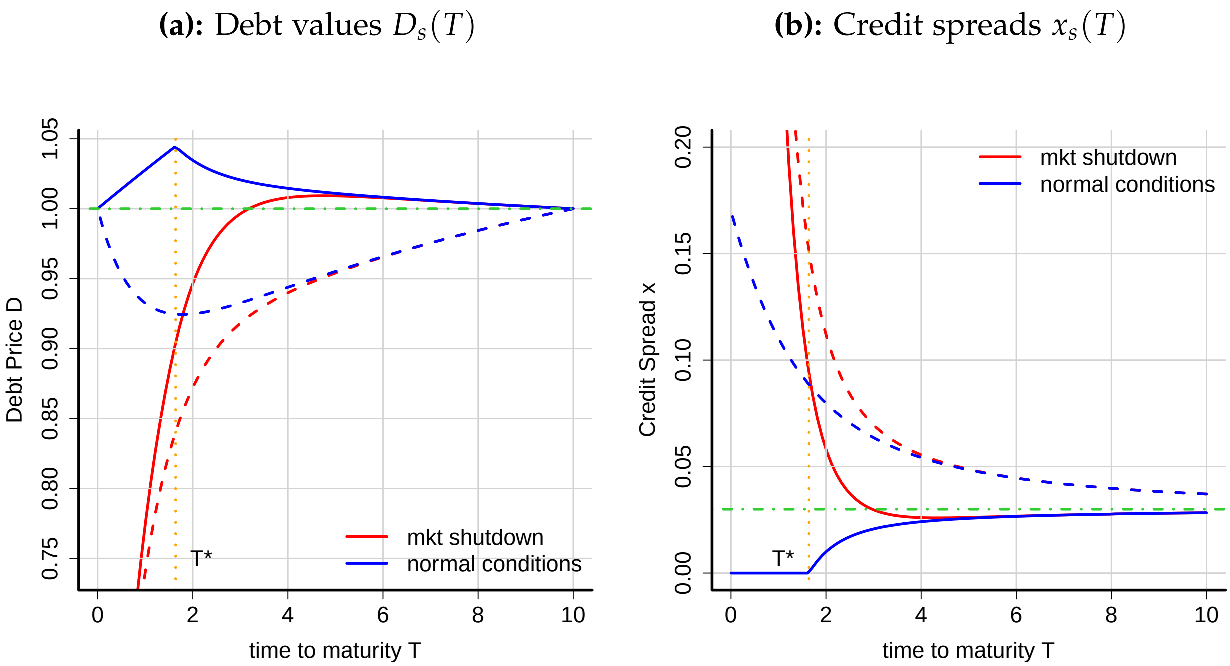
A comparison of the enterprise value in the callable vs. non-callable debt equilibrium allows us to quantify how much the ability to issue state-contingent securities is valuable for a firm,[16] whereas a comparison of the enterprise value in the callable vs. no-rollover risk equilibrium allows us to quantify how much of the rollover risk can be hedged out via callable debt. Appendix A.3.1 provides details on these alternative equilibria.

Figure 7 illustrates debt prices and credit spreads as a function of time-to-maturity T for a specific choice of parameters, when the firm rolls over make-whole callable debt with a make-whole spread ς, in the callable debt (solid lines), non-callable debt (dashed lines) and no-rollover (dot-dashed lines) equilibrium. In normal market conditions, bonds are issued at par $(D_1(m) = 1)$; in the callable debt equilibrium, investors anticipate perfectly that the firm will be refinancing its debt at the optimal time-to-maturity T^*, so that bond prices (resp. credit spreads) converge to the call price (resp. the make-whole spread ς) as $T \to T^*$. Instead, in a market shutdown, in both the callable and non-callable equilibrium, bond prices converge, as $T \to 0$, to their default value ρ. Absent rollover risk, the credit spread of the firm is simply equal to $\lambda(1-\rho)$ — i.e. the expected loss on the bond per unit of time.

Figure 8 instead shows equity and enterprise values, once again in the three equilibria of interest. In the callable debt equilibrium, the equity value is once continuously differentiable at $T = T^*$, a necessary condition for the optimality of the prepayment time. The prepayment option is value-enhancing for firm's shareholders; using prepayable debt also enhances enterprise value, defined as the sum of equity and debt values. Our setup is clearly an environment where the Modigliani-Miller theorem is violated: with capital markets' imperfections — here, the likelihood of a market shutdown— the enterprise value depends on the firm's financial policy— here, its refinancing strategy.

[16]When firms only use non-callable debt, one could wonder whether rollover risk can be mitigated by spreading their debt maturities. With the stark assumption we make in our model — any amount of debt to be rolled causes the firm to default during a market shutdown — outcomes are worse for firms spreading their debt maturities. If instead, default induced by rollover needs was a smooth function of the amount of debt to be rolled, there would indeed be a role for debt maturity smoothing, but the qualitative conclusion of the analysis in this section would remain valid.

Figure 7: Debt prices and credit spreads



Parameters: $r = 5\%$ p.a., $\lambda = 4.3\%$, $m = 10$ yrs, $y = 1$, $b = 5$, $\rho = 30\%$, $\varsigma = 0$, $1/\theta_u = 1$ yrs and $1/\theta_d = 5$ yrs. The resulting (equilibrium) par coupon is $c = 7.8\%$. Solid (resp. dashed) lines show debt price and credit spreads for the callable debt (resp. non-callable debt) equilibrium. The green dot-dash lines represent the debt price and credit spreads in the no-rollover risk equilibrium.

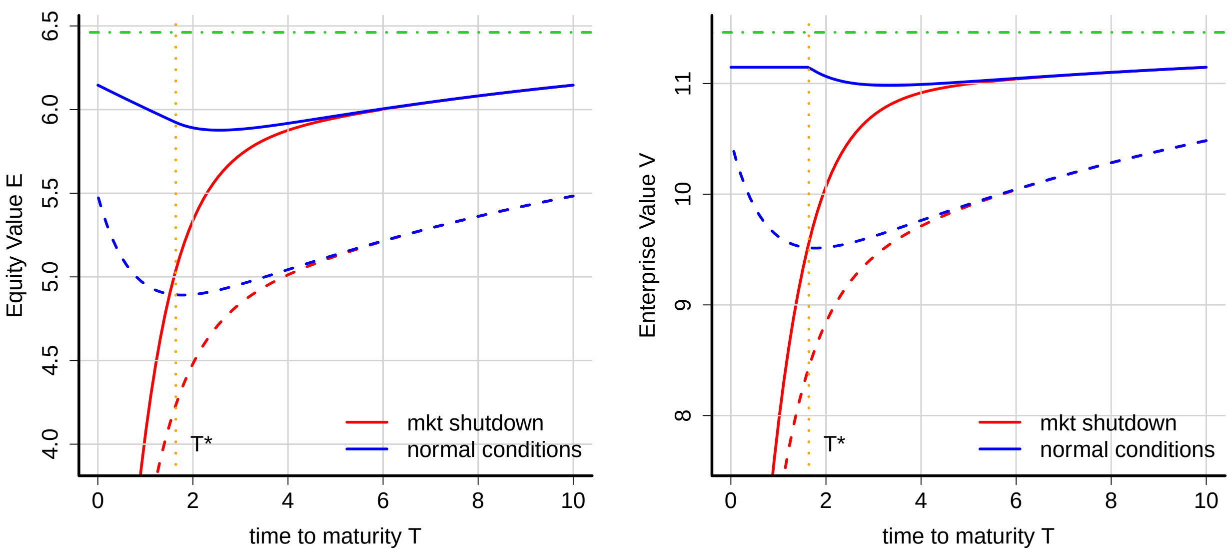
6.1.3 Creditor-shareholder agency conflicts

By reducing the likelihood of default due to market shutdowns, dynamic rollover risk hedging by firms' management is beneficial to equity values, but also to debt prices. Given that shareholders bear the cost of this dynamic hedging — via the payment of costly call premiums to debt holders — our model uncovers a new form of creditor-shareholder agency conflicts. This conflict is best seen when focusing on the optimal call time-to-maturity T^*, which is a declining function of the firm's debt-to-EBITDA b/y (left panel of Figure 9): the greater the firm's leverage, the greater the rollover risk hedging benefits accrue to creditors, the less shareholders are incentivized to pay a call premium to refinance early, the later they refinance (when markets are open) and the greater the probability of a default triggered by a market shut-down.

This creditor-shareholder agency conflict also impacts the dead-weight losses arising

Figure 8: Equity and enterprise values

(a): Equity values $E_s(T)$ **(b):** Enterprise values $V_s(T)$



Parameters: $r = 5\%$ p.a., $\lambda = 4.3\%$, $m = 10$ yrs, $y = 1$, $b = 5$, $\rho = 30\%$, $\varsigma = 0$, $1/\theta_u = 1$ yrs and $1/\theta_d = 5$ yrs. The resulting (equilibrium) par coupon is $c = 7.8\%$. Solid (resp. dashed) lines shows equity and enterprise value for the callable debt (resp. non-callable debt) equilibrium. The green dot-dash lines represent equity and enterprise value in the no-rollover risk equilibrium.

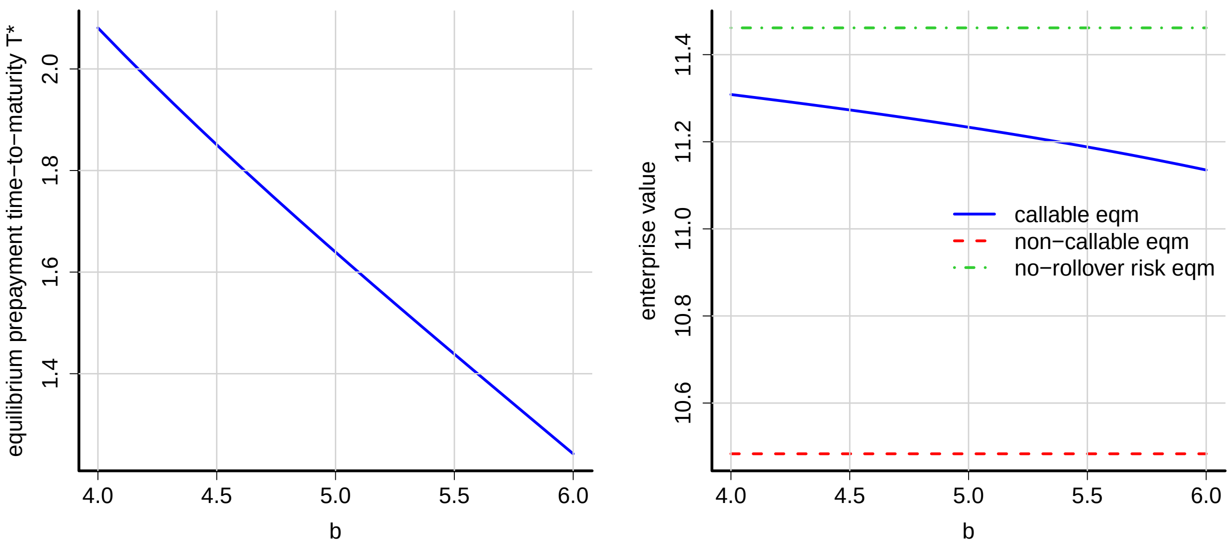
from default due to rollover risk, and how much of these losses can be mitigated by the dynamic call option exercise strategy. The right panel of Figure 9 illustrates this point: it shows enterprise value in the no-rollover risk, in the non-callable debt, and in the callable debt equilibria. With callable debt, enterprise value can get close to the "first best" (the no-rollover risk equilibrium) when debt is low, but creditor-shareholder agency conflicts reduce the benefits of callable debt when firm's leverage is high.

Lastly, one can also consider the optimal strategy that a manager maximizing *firm value* (rather than equity value) would use in order to refinance the firm's debt. Such strategy is trivial: the firm-value-optimizing manager would refinance continuously when credit markets are open.[17] This strategy is far from what shareholders end up

[17] This result follows from the assumption that there are no dead-weight costs upon debt issuance. In the presence of issuance costs, our analysis of Section 6.1 is almost identical. However, the optimal strategy for a firm-value-maximizing manager would also include an optimal refinancing time cutoff; the same

Figure 9: Debt overhang

(a): Time-to-maturity T^* vs. debt b **(b):** Enterprise value $V_1(m)$ vs. b



Parameters: $r = 5\%$ p.a., $\lambda = 4.3\%$, $m = 10$ yrs, $y = 1$, $\rho = 30\%$, $\varsigma = 0$, $1/\theta_u = 1$ yrs and $1/\theta_d = 5$ yrs. Left plot shows optimal time-to-maturity T^* when b varies. Right plot shows the enterprise value $V_1(m)$ (a) in the callable debt equilibrium (solid blue), (b) the non-callable debt equilibrium (red dashed) and (c) the no-rollover risk equilibrium (dotted green), as a function of b.

doing — only refinancing once the remaining time-to-maturity falls below T^*. The difference between the firm-value-optimizing and equity-value optimizing managers is another illustration of the creditor-shareholder agency problem present in our model.

6.2 Generalization

The model developed in Section 6.1 is simple by design, as it is meant to demonstrate how firms can mitigate rollover risk during market shut-downs by proactively refinancing their callable debt when credit market conditions are benign. In order to more carefully quantify the costs and benefits incurred by firms when hedging rollover risk, we need to expand our model to take into account the other key driver of firms' prepayment

creditor-shareholder agency conflict would make the firm-value-maximizing manager refinance earlier than the equity-value-maximizing manager.

decisions: the interest rate and spread option that equity holders are long when issuing fixed-rate prepayable corporate debt. To do this, we now consider an environment where interest rates and default intensities are time-varying.

Let s_t be a discrete state continuous time Markov chain with generator matrix Θ, taking values in $\{1, ..., n\}$. The firm's default intensity is equal to $\lambda_t = \lambda(s_t)$. The short term interest rate is equal to $r_t = r(s_t)$. The firm's net income is equal to $y_t = y(s_t)$. As in Section 6.1, the firm rolls over its debt with notional amount b, maturity m and a coupon rate c_t whose value depends on the state prevalent at the time the debt was refinanced last. Capital markets are imperfect: whenever in state s_t, when the firm attempts to issue debt, it will succeed with probability $\pi(s_t)$ and fail with probability $1 - \pi(s_t)$, in which case the firm is forced to default.

Shareholders solve a problem similar to (4): they choose a refinancing strategy, balancing the benefits of refinancing the firm's debt early (which allows the firm to lock a low interest expense for the next m years at least, and also allows it to mitigate rollover risk) vs. its cost — equal to a call premium $\kappa(s_t, T_t, c_t)$ (per unit of outstanding debt) that the firm has to pay if it decides to refinance a bond with coupon c_t and remaining time-to-maturity T_t, when the aggregate state of the economy is s_t.[18]

Since we focus on an environment in which firms only issue par-priced debt, for each aggregate state s at which a firm issues a new m-year maturity bond, there is a corresponding (endogenous) coupon $\mathcal{C}(s)$ that insures that such debt is sold at par to investors. The relevant state for a firm is thus the triplet (i, j, T), encoding the current (exogenously evolving) aggregate state $i \in \{1, ..., n\}$, the aggregate state j that was prevalent at the last refinancing time (and thus the current debt coupon is $c = \mathcal{C}(j)$), and the bond time-to-maturity $T \in [0, m]$. The firm's equity value solves

$$E_{ij}(T) := \sup_{\tau} \quad \mathbb{E}_{i,j,T}\left[\int_0^{\tau_d} e^{-\int_0^t r(s_u)du}\left[\left(y(s_t) - c_t b\right)dt - b\kappa\left(s_{t-}, T_{t-}, c_{t-}\right)dN_t^{(\tau)}\right]\right]$$

subject to $\quad dc_t = \left(\mathcal{C}(s_t) - c_{t-}\right)dN_t^{(\tau)},$

[18]The call premium not only depends on the bond's remaining time-to-maturity T, but also on the current coupon of the firm's debt and the current aggregate environment: with make-whole callable debt, the call premium is increasing in the coupon rate and decreasing in the level of interest rates, while with fixed-price callable debt, the (initial) call premium is typically either 100%, or 50% of the coupon rate (see Powers, 2021), decreasing as time-to-maturity decreases, and independent on the level of interest rates.

$$dT_t = -dt + (m - T_{t-})\, dN_t^{(\boldsymbol{\tau})},$$

where we have noted $N_t^{(\boldsymbol{\tau})}$ the counting process for prepayment events under policy $\boldsymbol{\tau}$. Debt investors take into account shareholders' prepayment strategy. When the firm defaults, they get paid $\rho(s_t) \in (0,1)$ per unit of principal, where the (possibly state-dependent) recovery rate $\rho(s_t)$ is linked to the liquidation value of the firm's project $\alpha(s_t)V_0(s_t)$, and $V_0(s_t)$ is the unlevered, risk-free project value. The bond price satisfies

$$\begin{aligned} D_{ij}(T) = \mathbb{E}_{i,j,T} \Bigg[\int_0^{\tau_r \wedge T} & e^{-\int_0^t (r(s_u)+\lambda(s_u))du} \left(\mathcal{C}(j) + \lambda(s_t)\rho(s_t)\right) dt \\ & + e^{-\int_0^{\tau_r \wedge T} (r(s_u)+\lambda(s_u))du} \left[1_{\{\tau_r \le T\}} \left(1 + \kappa\left(s_{\tau_r}, T_{\tau_r}, \mathcal{C}(j)\right)\right) \tilde{\omega}_{\tau_r} + 1_{\{T \le \tau_r\}} \right] \Bigg], \end{aligned}$$

where $\tilde{\omega}_t$ is a random variable that is equal to 1 with probability $\pi(s_t)$ and equal to $\rho(s_t)$ with probability $1 - \pi(s_t)$. The definition of an Equilibrium of our economy is a simple extension of Definition 1. In Appendix IA.2, we write down the variational inequality satisfied by the equity value $E_{ij}(T)$, as well as the Feynman-Kac equation satisfied by the debt price $D_{ij}(T)$. The key decision variable for the firm's managers is a set of optimal prepayment times-to-maturity T_{ij}^* when the aggregate state is $s_t = i$ and the firm is currently paying coupon $c_t = \mathcal{C}(j)$. These times satisfy a set of conditions similar to (7) — one condition per aggregate state and coupon state.

7 Quantitative evaluation and counter-factuals

In this section, we estimate our model of corporate debt refinancing decisions, confront some of its equilibrium outcomes to their data counterpart, and study quantitatively various counter-factuals.

7.1 Model estimation

Our discrete state continuous time Markov chain s_t drives the behavior of interest rates r_t, default intensities λ_t, recovery rates ρ_t, EBITDA y_t, and rollover risk π_t. For simplicity, we assume that debt-to-EBITDA b/y_t is constant and equal to b/y, calibrated to match

HY firms' average debt-to-EBITA ratio in our sample of focus. Recovery rates are also set constant and equal to $\rho_t = \rho = 30\%$, following Moody's.[19] We choose a contractual debt maturity $m = 10$ years, roughly corresponding to the empirical average maturity in our HY bond sample. Consistent with our empirical evidence, we assume that bonds are callable by firms at a call premium κ that reflects a hybrid structure — callable at a make-whole price (with make-whole spread of zero) for the first half of contractual life, and then at a fixed price linearly decreasing during the second half of contractual life (from par plus one year worth of coupon to par at maturity). We then compute the values $\{\lambda(s), r(s)\}_{s\leq n}$ and the transition intensities $\{\theta_{ij}\}_{1\leq i,j\leq n}$ of our discrete state space model so that the stochastic processes $\lambda(s_t)$ and $r(s_t)$ approximate one-factor square root diffusion processes (as in Cox, Ingersoll Jr and Ross, 1985) estimated using maximum likelihood, and under the assumption that rates and default intensities are orthogonal.[20] Since the market-shut down risk is complex to estimate in the data, we assume that firms cannot roll over their debt if the default intensity is above a certain threshold $\bar{\lambda}$, and we study the sensitivity of our results to the choice $\bar{\lambda}$. We solve our model using a standard finite-difference method; the optimal stopping problem is solved by numerically encoding the variational inequality solved by the equity value as a linear complementarity problem (Huang and Pang (2003)). Table 4 summarizes the parameter values for our baseline model.

Our parametrization leads to average credit spreads for non-callable debt, in the absence of credit market shut-down risk, of $(1-\rho)\mathbb{E}[\lambda_t] = 5.50\%$ — which corresponds to the average HY CDX spread over our sample period. The cutoff $\bar{\lambda}$ is chosen such that the ergodic percentage of time spent in the sudden-stop state is equal to 5%.

[19] See Moody's Ultimate Recovery Database. According to this study, bonds' recovery rates over a long time period ending in 2007 have averaged 37%, with a median of 24%. We thus choose the intermediate value of $\rho = 30\%$ in our analysis.

[20] Our model of the short rate is estimated using time-series data for T-bill rates from 1990 to 2015, while our model of default intensities is estimated using time-series data for the high-yield CDX index, from 2004 to 2020, under the assumption that recovery rates are constant and equal to 30%. For the time-period 2002-2020, the correlation between the high-yield CDX index and the 10-year zero coupon treasury rate is equal to 1%, hence our assumption that these two processes are orthogonal in the model.

Parameter	Value	Interpretation
$\mathbb{E}[r_t]$	0.038	Long-run mean of r_t
$\sqrt{\text{Var}[r_t]}$	0.02	Long-run standard deviation of r_t
$\tau_{0.5}(r_t)$	4.39	Half life of process r_t, in years
$\mathbb{E}[\lambda_t]$	0.079	Long-run mean of λ_t
$\sqrt{\text{Var}[\lambda_t]}$	0.029	Long-run standard deviation of λ_t
$\tau_{0.5}(\lambda_t)$	1.82	Half life of process λ_t, in years
b/y	4	Debt-to-EBITDA ratio
m	10	Debt maturity, in years
ρ	0.3	Recovery rate
$\bar{\lambda}$	0.13	Default intensity beyond which rollover is not possible

Table 4: Baseline parameter values

7.2 Model solution

The solution of our model is a set of optimal prepayment times-to-maturity $\{T^*_{ij}\}_{1\leq i,j\leq n}$ for firms with coupon $c_t = \mathcal{C}(j)$ when the state of the economy is $s_t = i$. In Figure 10 we plot such prepayment times as a function of the level of default intensities for the model with and without rollover risk. Absent rollover risk, T^* is a globally monotone decreasing function of default intensities, as the interest cost savings incentive is weaker when credit spreads are elevated; instead, in the presence of rollover risk, firm's management also refinances when default intensities are elevated, as the likelihood of a sudden stop becomes more likely. We plot in Internet Appendix Figure IA-5 the optimal prepayment time-to-maturity as a function of rates and of coupons; firms delay their refinancing decisions when the current bond coupon is low or current interest rates are elevated, a natural consequence of the interest cost savings' channel.

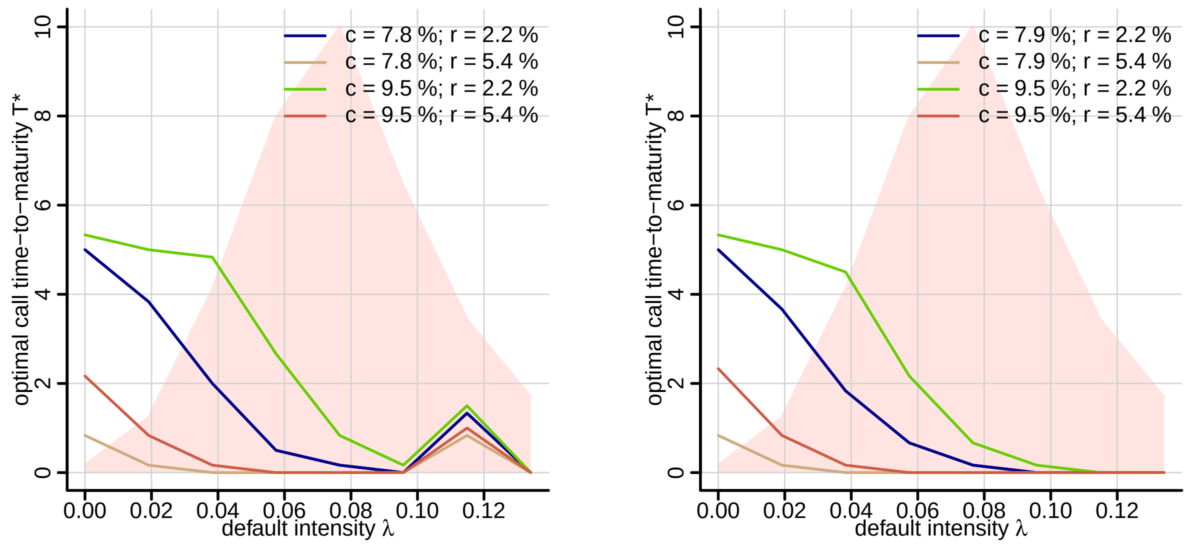
7.3 Model validation

In order to assess the performance of our model, we study its predictions for prepayments and prices, and compare such predictions to their data counterpart.

We first note that, for the purpose of determining firms' optimal prepayment behav-

Figure 10: Optimal call policies

(a): T^* with rollover risk **(b):** T^* without rollover risk



Both plots show optimal call time-to-maturity T^* as a function of the default intensity λ_t for various levels of coupon and interest rates. Left (resp. right) hand side shows such policies in the model with (resp. without) rollover risk.

ior, we used default intensities estimated under the risk-neutral measure — by leveraging the dynamics of CDS prices. However, actual default realizations are observed under the statistical measure. Thus, when focusing on model predictions for prepayments, we simulate our model assuming that under the statistical measure, defaults never occur.[21] Alternatively, one can interpret this assumption as if firms' default realizations occur according to the risk-neutral measure, and after each default, the bankrupt firm disappears and an identical firm enters our economy with the same time-to-maturity T and the same bond coupon c. This modeling device allows us to generate a long-run distribution $f_{ij}(T)$, corresponding to the time-series average percentage of time a firm spends in state s_i with coupon $\mathcal{C}(j)$ and with time-to-maturity in $[T, T + dT]$. See Appendix IA.2.2 for technical details.

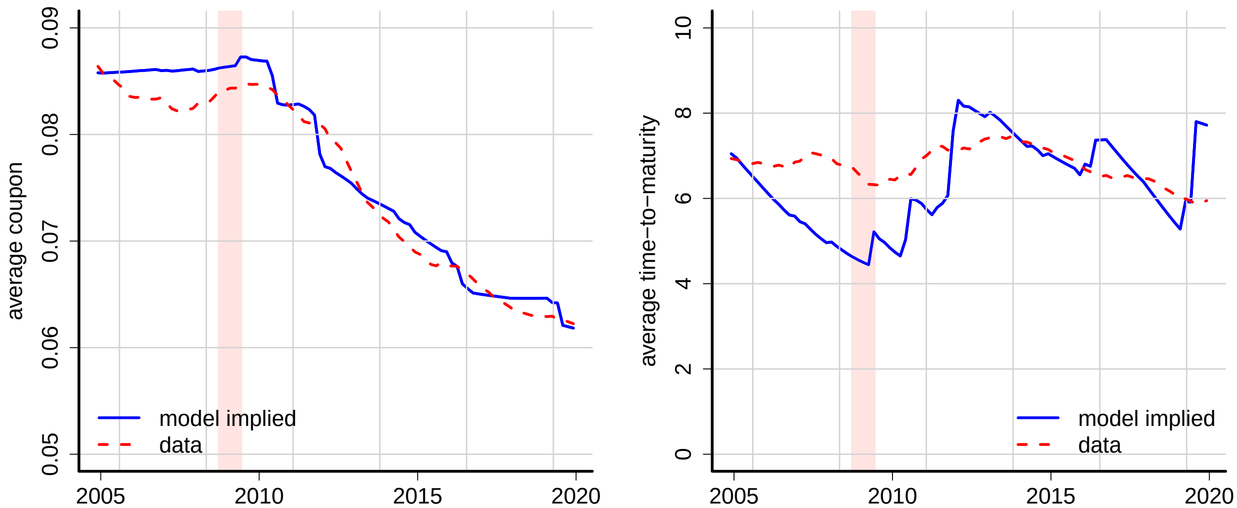
[21] In particular we are assuming that defaults neither occur due to the default intensity process $\lambda(s_t)$, nor due to the realization of a sudden stop.

Using this modeling device, we study, through the lens of our model, the sample period 2004-2020; given the observed empirical distribution of firms over their coupon c and time-to-maturity T in 2004 (Internet Appendix Figure IA-6), we feed the time path of realized interest rates and default intensities (Internet Appendix Figure IA-7), and produce a model-implied time-series of average coupons and average bond time-to-maturity, which we can then compare to the data, as Figure 11 illustrates. The model produces a trajectory of (cross-sectional) average bond coupons that is consistent with the data for HY corporate bonds during that time period: broadly declining with the level of interest rates, from a high of 8.5% in 2004 to an all-time low just above 6% in 2020. The model produces consistent — albeit more volatile — (cross-sectional) average bond time-to-maturity, with periods of decreases as the economy suffers a sudden stop (during the 2008-2009 financial crisis), and increases as firms prepay and early-refinance their debt, taking advantage of lower levels of interest rates. The time-to-maturity trajectories are a direct result of the optimal refinancing decisions made by firms in our model; the related average prepayment rate over that time period is 8.6%, i.e. somewhat lower than the 11.7% average prepayment rate observed in the data over that time period. In the Internet Appendix, we show in Figure IA-8 the trajectory of the cross-sectional standard deviations for coupons and time-to-maturity. The model-implied standard deviations are generally lower than those observed in the data, since (a) prepayment events in the model are entirely explained by the interest cost savings and rollover risk management motives, and (b) the only source of cross-sectional heterogeneity in our model is the time at which firms first issued debt (firms' default intensities are perfectly correlated).

Our model also generates callable bond prices and endogenous par coupons $\mathcal{C}(s)$ (for all states $s \leq n$) for newly issued callable bonds, which we can compare to hypothetical newly issued non-callable bonds in that same callable debt equilibrium. The ranking of par coupons for callable and non-callable debt is non-trivial: on one side, callable bonds' investors are short a call option, which tends to push the par yield of callable bonds above those of non-callable bonds; on the other side, when callable bonds are called, investors receive a premium above and beyond the coupon compensation, which tends to push pay yields of callable bonds below that of non-callable debt. We plot the par yield spreads to the risk-free benchmark as a function of rates and default intensities prevalent at the time of the bond issue in Figure IA-9; these computations illustrate

Figure 11: Model vs. data: means

(a): Average coupon **(b):** Average time-to-maturity



Left (resp. right) hand side shows the firms' average coupon (resp. time-to-maturity) in the data (in dash red) and implied by our model (in solid blue) for the time period 2004-2020. Shaded pink area indicates sudden stop periods.

that par yield for both callable and non-callable debt are similar, with no systematic ranking of these debt compensation measures. This analysis is also supported by our empirical calculations: Figure IA-4 shows, for our bond sample, the difference between (1) a callable bond coupon at issuance, and (2) its theoretical non-callable par coupon counterpart using our synthetic non-callable construct from Section 5.2.2.

7.4 The costs and benefits of rollover risk management

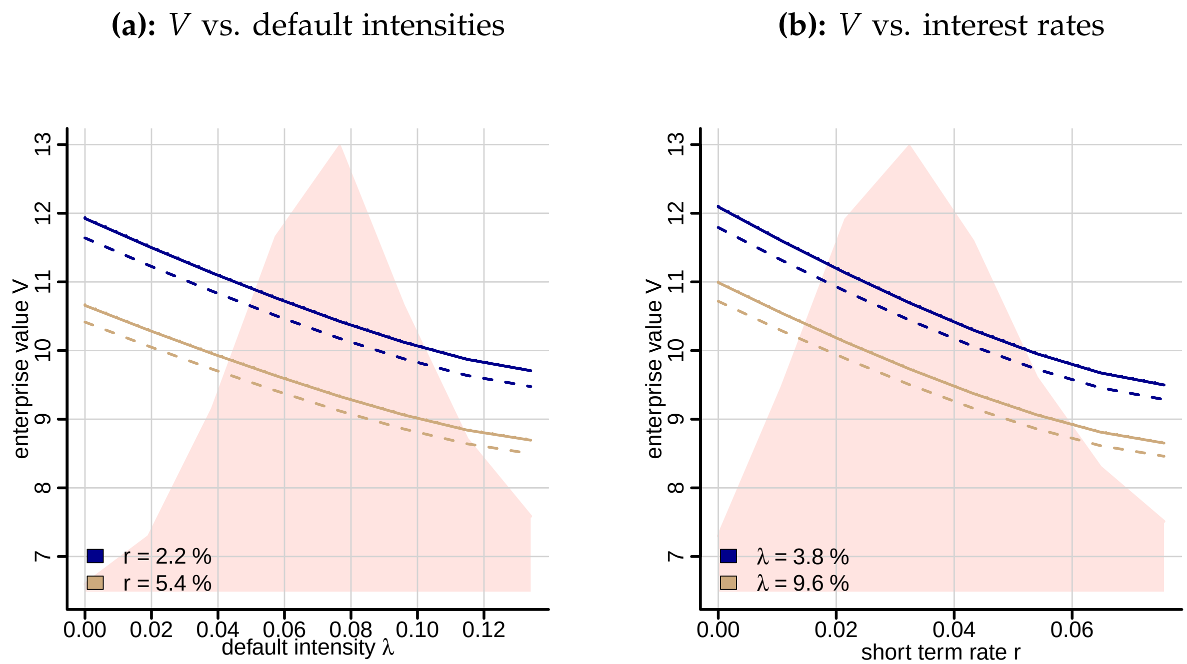
Our structural model of the firm allows us to compute counter-factuals that would not be available with reduced form empirics.

First, we can compute the amount of dead-weight losses due to rollover risk that firms avoid by actively managing their callable bonds' effective maturities, by comparing enterprise value in the callable and non-callable equilibria and comparing such values to those in the no-rollover risk equilibrium. Figure 12 illustrates these computations. The cost of rollover risk — measured as the difference in enterprise value between the no-rollover risk and non-callable debt equilibria — varies state by state, but is approximately 4% of enterprise value; allowing the firm to use callable debt and to strategically exercise such call options almost entirely restores the first best enterprise value. This counterfactual highly depends on the likelihood of a sudden stop event; in Figure IA-10, we show an identical analysis when the sudden stop occurs at default intensities greater than $\bar{\lambda} = 11\%$, corresponding to an ergodic percentage of time spent in the sudden-stop state equal to 15% — i.e. 3 times more likely than in our baseline analysis. In that case, rollover risk is more costly to firms, and callable debt does not fully bring enterprise value to its level absent sudden stop risk.

Our analysis suggest however that callable debt is an effective — but imperfect — tool to mitigate default risk induced by credit market shut-downs. This observation begs the question as to why monetary policy authorities — for instance the Federal Reserve during the COVID crisis — feel the need to intervene in credit markets when instead, the private sector has already engineered a device that allows it to mitigate almost entirely such risk. Our results suggest that this type of intervention might have only limited benefits (in terms of reducing credit losses arising from the sudden stop), with potentially non-trivial costs (un-modeled in this paper, but that would naturally arise from the moral hazard consequences of a central bank backstop of credit markets).

We can also quantify the extent to which rollover risk (a) increases prepayment rates in HY credit markets, and (b) alters the average debt interest expense incurred by firms rolling over callable debt. Table 5 suggests that rollover risk induces firms to prepay their callable debt at 25% higher frequency (in our baseline calibration with moderate rollover risk) than the frequency prevalent in an environment without rollover debt; while average coupons paid by firms are roughly the same in the two environments, rollover risk causes firms' shareholders to spend 20bps p.a. more in call premium payments in order to mitigate such risk.

Figure 12: Enterprise value



Left hand side shows enterprise value at the time of a new bond issuance as a function of the default intensity λ for various levels of interest rates for the callable debt equilibrium (solid lines), the non-callable debt equilibrium (dashed lines) and the no-rollover risk equilibrium (dot-dashed lines); right hand side instead shows these same enterprise values as a function of the short rate r for various levels of default intensities.

Ergodics with callable debt	no rollover risk	moderate rollover risk	high rollover risk
% of time spent in sudden stop	0%	5%	15%
average prepayment rate	6.1% p.a.	7.6% p.a.	9.7% p.a.
average call premium paid	1.2% p.a.	1.4% p.a.	1.9% p.a.
average debt coupon	8.4% p.a.	8.4% p.a.	8.4% p.a.

Table 5: Ergodic statistics

8 Conclusion

This paper provides a comprehensive analysis of the role of callable bonds in managing the term structure of debt maturities for high-yield firms in the US corporate bond market. We argue that firms strategically utilize callable debt to proactively mitigate their exposure to credit market shutdown risk, thereby ensuring their continued access to capital and reducing the likelihood of failure when debt obligations come due.

We find that high-yield firms systematically rely on debt with various call protection structures and, using a new measure of option moneyness, document that firms efficiently exercise interest rate and credit spread options embedded in their bonds. Holding constant the interest cost savings incentives, we show that firms have a significantly greater probability of early refinancing their callable bonds if rollover risk is high.

We formalize the interest cost savings and rollover risk management channel through a theoretical framework that enables us to compute counterfactual scenarios and quantify the costs incurred by firms in hedging against credit market shutdown risk. When sudden stop risk is moderate, the ability to use callable debt almost entirely dissipates dead-weight losses from rollover risk. Greater corporate leverage, however, reduces the effectiveness in sudden stop risk hedging, due to creditor-shareholders agency conflicts.

Overall, our findings contribute to the understanding of how high-yield firms actively manage their debt maturities to navigate credit market uncertainties. By strategically utilizing callable bonds, firms can enhance their financial resilience, maintain access to capital, and mitigate the risk of default during periods of market stress.

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Appendix

A.1 Additional figures and tables

Table A-1: Variable Descriptions

Variables	Data Source	Description
Bond variables:		
Prepaid (0/1)	Mergent FISD	Indicator tells if a bond has been prepaid before its contractual maturity date.
Coupon rate (%)	Mergent FISD	Bond annual coupon rate.
Swap rate (%)	FRED	Interpolated swap rate based on time to maturity.
Bond rating	Mergent FISD	The numeric value of bond's rating. AAA = 1, AA = 2, etc. When multiple ratings are available from different rating agencies at the same time, we use the average of all numerical rating values.
Year(s) to maturity	Mergent FISD	Number of years until bond maturity date.
Issuance amount ($M)	Mergent FISD	Bond offering amount at issuance.
Call type	Mergent FISD	Based on the type of embedded call option, a bond is classified as either 1) Non-callable; 2) Fixed price callable; 3) make-whole only callable; or 4) hybrid.
Bond trading price	TRACE	Median of bond trading price in the given month.
Swap spread (%)	TRACE, FRED	Difference between implied yield-to-maturity based on the bond trading price and interpolated swap rate.
CDS (%)	IHS Markit	Interpolated result of the CDS spreads term structure.
CDS basis(%)	Self-calculated	The median difference between interpolated CDS and yield-to-maturity of non-callable and make-whole only callable bonds within each rating category (IG vs HY)
Moneyness (%)	Self-calculated	Moneyness of call provision scaled by the strike price.
Outstanding ($M)	Mergent FISD	The outstanding balance at given date.
Call status	Mergent FISD	Indicator tells if a bond is callable, and which type of call provision is effective at the given date.
Issuer variables:		
Total asset ($M)	Compustat	Total assets in million USD.
Debt-to-Asset Ratio	Compustat	Total debt to asset ratio.
Debt-to-EBITDA Ratio	Compustat	Total debt to trailing 12 month EBITDA ratio.
Rollover Risk	Compustat or Mergent FISD	Indicator if proportion of short term debt (due within 1 year) exceed certain cutoff point.

Table A-2: HY senior unsecured bonds with and without CDS information

This table shows summary statistics for our sample of HY senior unsecured bonds. Both panels report statistics on bond and issuer characteristics (at the time of issuance) at the bond-level. Panel A shows summary statistics for sub-sample with CDS information available. Panel B shows summary statistics for sub-sample without CDS. All variables are defined in detail in Appendix Table A-1.

Panel A: Bonds with CDS information

Variables	N	Mean	SD	P05	P25	Median	P75	P95
Bond characteristics:								
Coupon rate (%)	2,004	7.6	2.0	4.5	6.1	7.6	9.0	10.8
Bond initial rating	2,004	B+	-	BB+	BB	BB-	B	CCC
Year(s) to maturity	2,004	9.3	4.3	5.0	8.0	9.9	10.0	12.0
Issuance amount ($M)	2,004	501	447	8	225	400	624	1,329
Issuer characteristics:								
Total asset ($M)	1,936	15,188	31,927	1,135	2,929	7,280	16,528	41,099
Debt-to-Asset Ratio (%)	1,808	48.9	28.7	18.2	32.3	45.9	58.8	91.6

# bonds by call type:					
Call type:	All	Fixed price	Make whole	Non callable	Hybrid
N	2,004	394	363	281	966

Panel B: Bonds without CDS information

Variable	N	Mean	SD	P05	P25	Median	P75	P95
Bond characteristics:								
Coupon rate (%)	2,730	8.1	2.3	4.5	6.2	8.1	9.8	11.9
Bond initial rating	2,730	B+	-	BB+	BB-	B	B-	CCC
Year(s) to maturity	2,730	8.7	2.4	5.0	8.0	8.1	10.0	10.2
Issuance amount ($M)	2,730	381	332	100	175	300	500	1,000
Issuer characteristics:								
Total asset ($M)	2,471	6,046	15,149	299	1,013	2,242	5,752	18,651
Debt-to-Asset Ratio (%)	2,357	50.2	25.4	18.3	34.8	47.0	60.8	92.2

# bonds by call type:					
Call type:	All	Fixed price	Make whole	Non callable	Hybrid
N	2,730	981	140	118	1,491

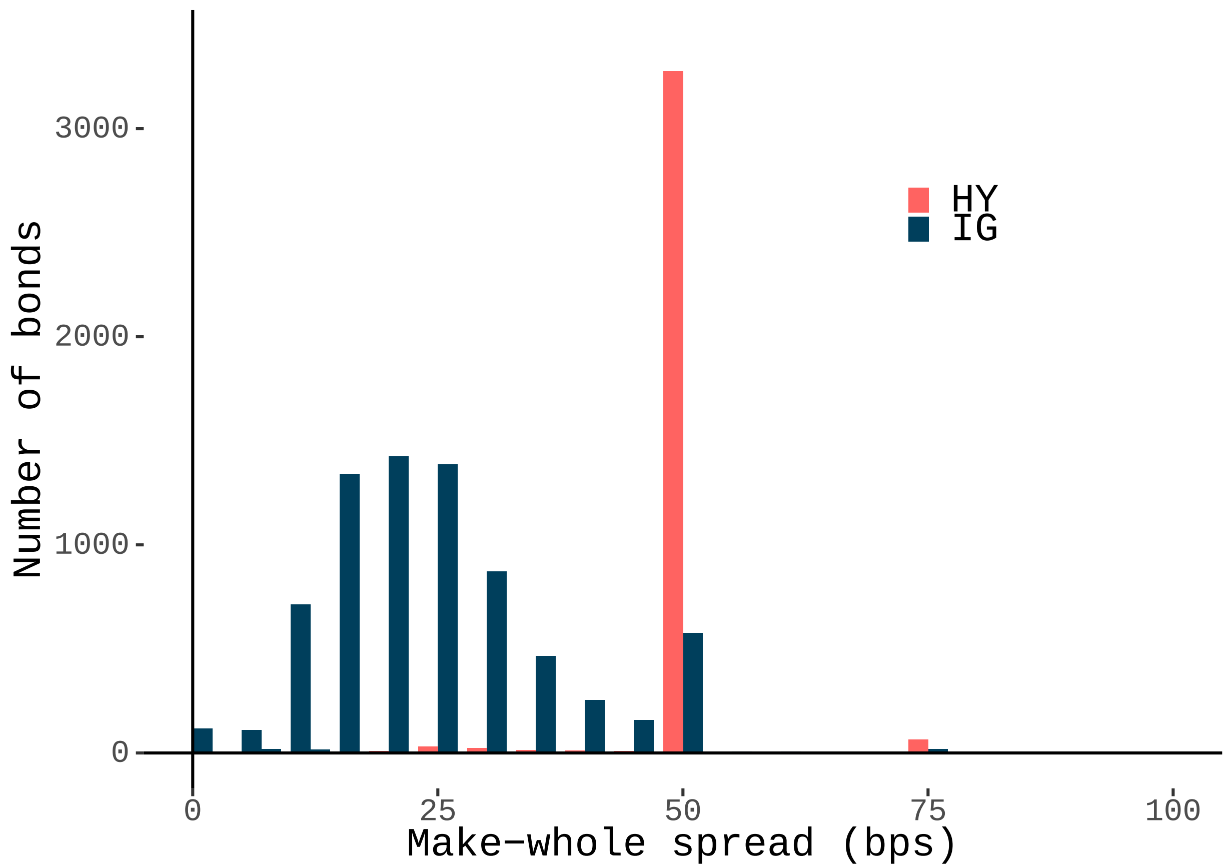


Figure A-1: Make-whole spread distribution
The figure displays the make-whole spread for hybrid and pure make-whole bonds in our sample.

Panel A: Non-investment grade bonds

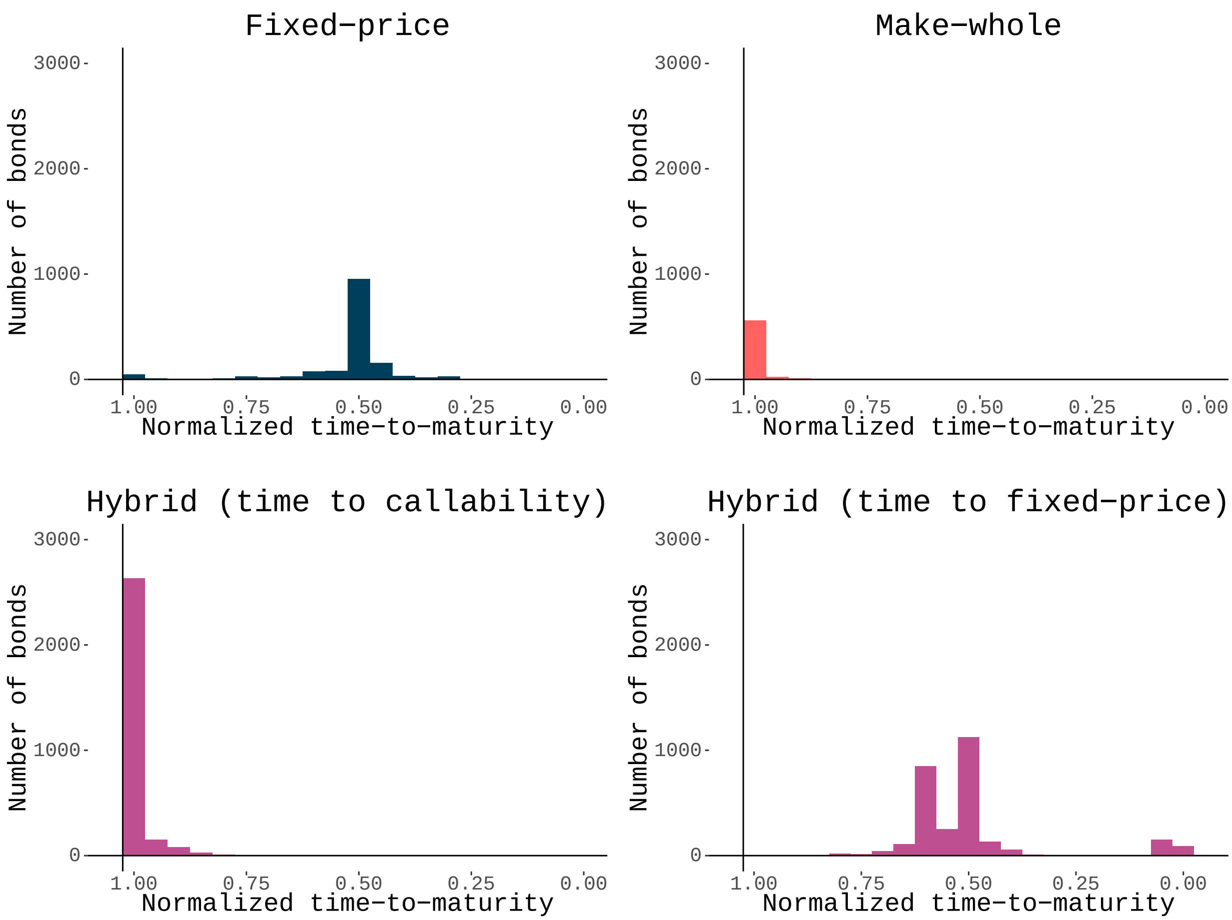


Figure A-2: When do bonds become callable?

Panel B: Investment grade bonds

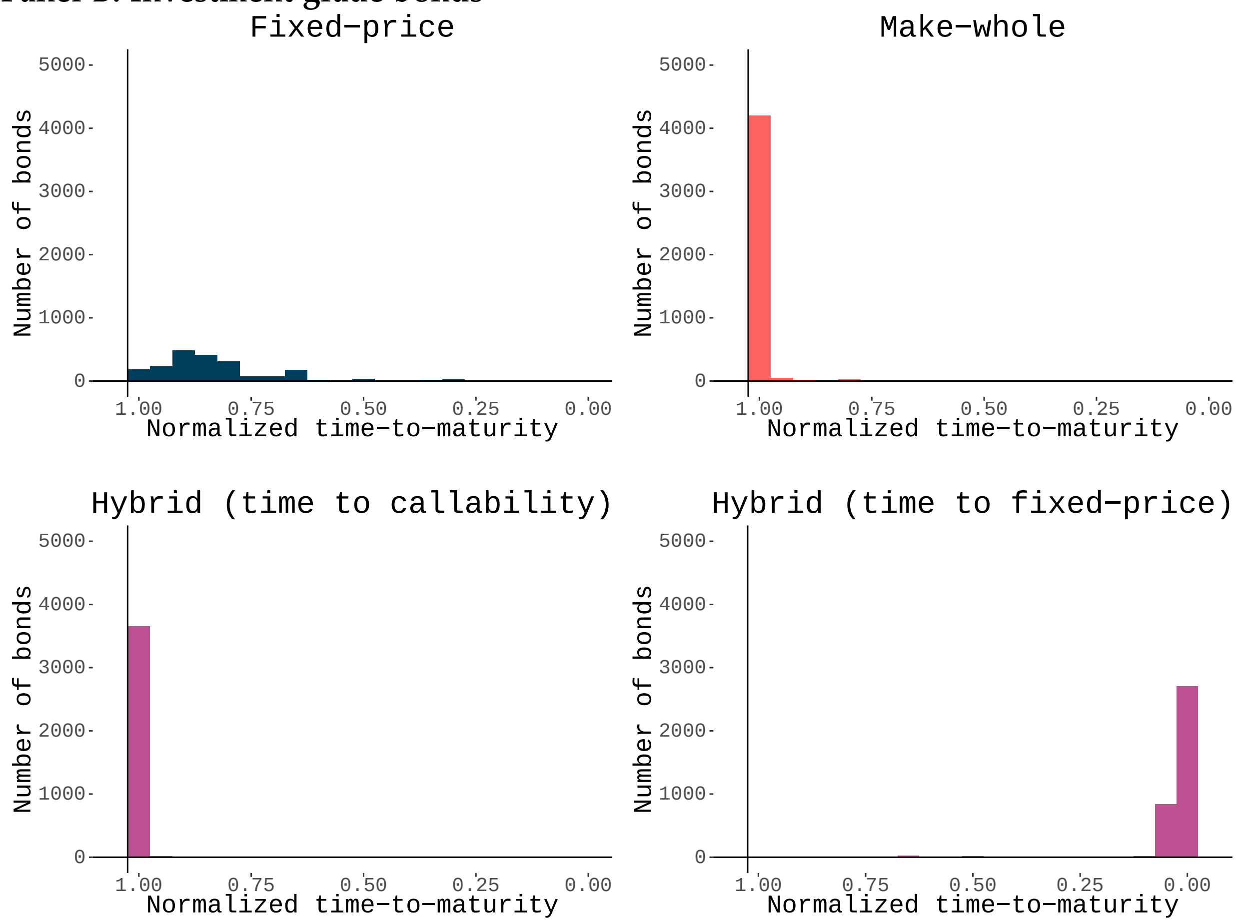


Figure A-2: When do bonds become callable? (continued)

Panel A (Panel B) of this figure shows when HY (IG) bonds become callable. The figure distinguishes between bonds that are callable at a fixed price, bonds callable with a make-whole provision, and "hybrid bonds", i.e., bonds that have both make-whole and fixed-price call provisions. For hybrid bonds, the figure additionally shows when the bonds switch from being make-whole callable to being fixed-price callable (bottom right in each panel). The x-axis is normalized by the contractual bond time to maturity at issuance for all plots in this figure.

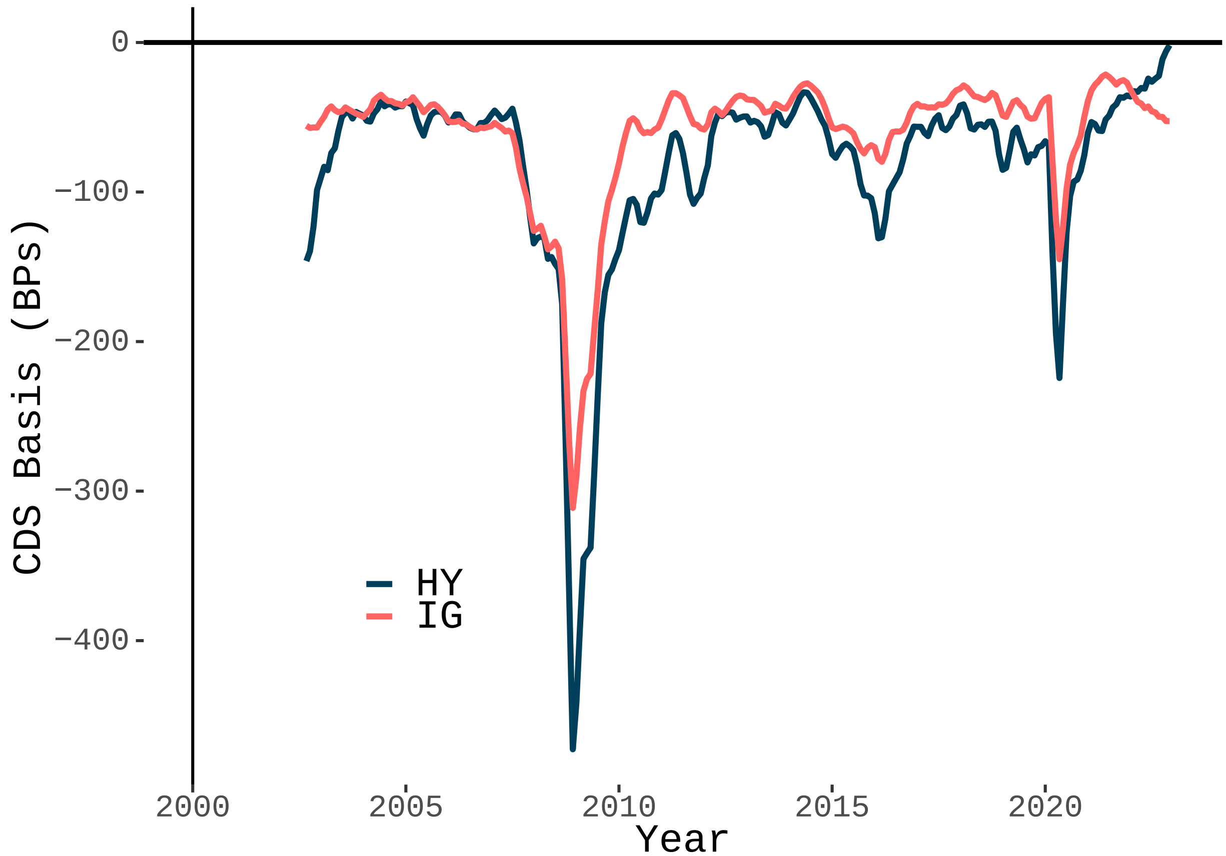


Figure A-3: CDS-bond basis
This figure shows the (three-month moving-average) CDS-bond basis for the IG and HY segment over time. The CDS-bond basis is defined as the median difference between the *CDS spread* for bonds in rating cluster k and the corresponding bond *swap spread* (i.e., yield-to-maturity minus swap rate). Only non-callable bonds (and pure make-whole callable bonds) are used in the CDS-bond basis calculation.

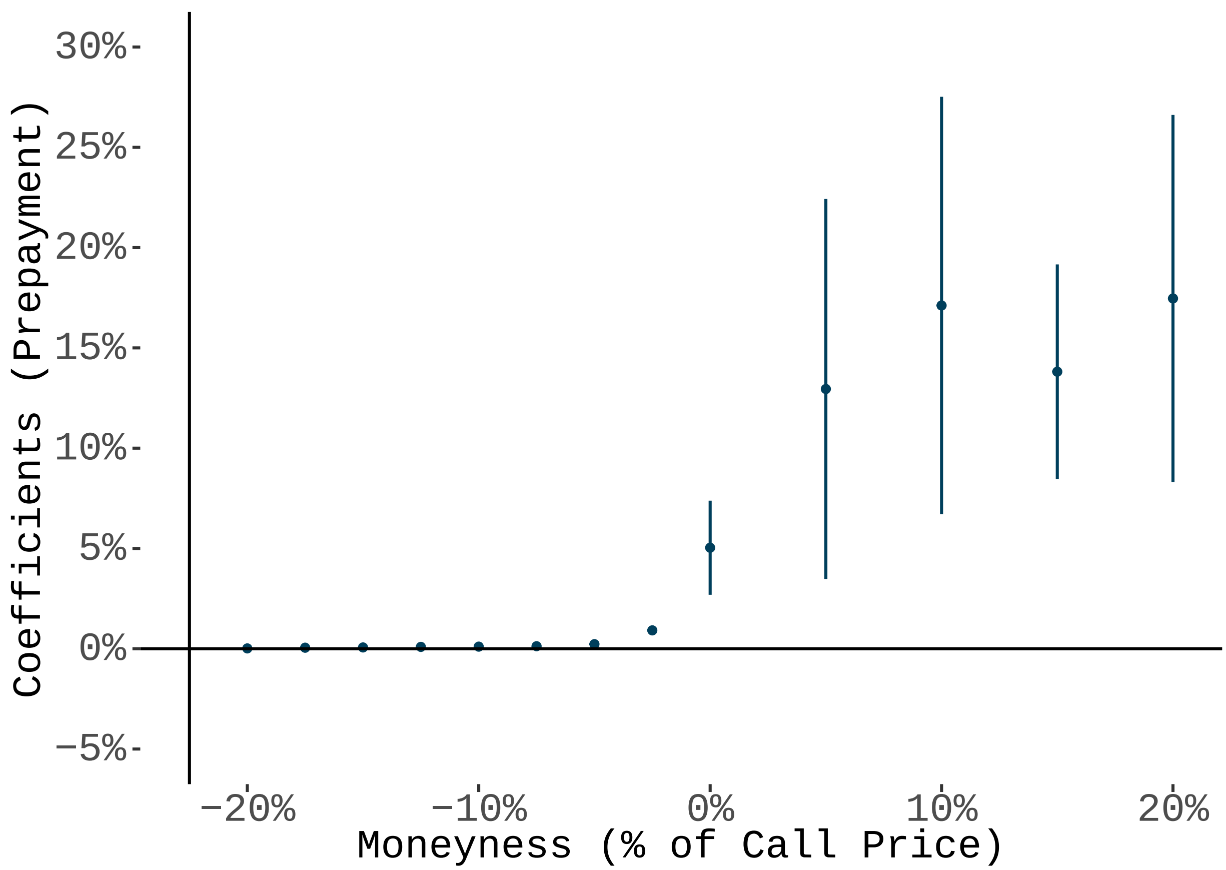


Figure A-4: Option moneyness and call option exercise probability
This figure shows regression coefficients of an estimation of bond prepayment probabilities by option moneyness. The omitted category is moneyness < 20%. The sample includes investment-grade bonds only. The figure shows 95% confidence intervals around the coefficient estimates.

A.2 Data construction

This section describes (a) the filters we apply when selecting the bonds contained in our final sample and (b) how we reconstruct the historical "life" of a particular bond. Details on how we link Mergent to Compustat are provided in the Internet Appendix.

A.2.1 Sample selection

We start with all bonds in the Mergent database. We then go through the following selection criteria:

1. We restrict the sample to corporate bonds, including: Corporate Debentures ("CDEB"), Corporate Medium Term Notes ("CMTN"), Corporate Pass Through Trusts ("CPAS"), Retail Notes ("RNT"), and Corporate Payment-in-Kind Bonds ("CPIK").
2. We exclude preferred bonds and restrict the sample to bonds issued by US firms that are outstanding at any point during our sample period (2000 - 2022).
3. We require that each bond must be included in the Mergent "Amout_outstanding" table, so we can track its outstanding amount over time.
4. We require that we can match the bond issuer to S&P's Compustat North America database. The matching process is described in the Internet Appendix.
5. We exclude bonds issued by financial and utilities firms (SIC codes 4,900 - 4,999 and 6,000 - 6,999).
6. We require that bonds have available information for at least three consecutive years.
7. We exclude convertible bonds and exchangeable bonds.
8. We carefully check frequent issuers and exclude bonds issued by financial subsidiaries of industrial firms (ie. bonds issued by Ford Credit or Caterpillar Financial Service).
9. We exclude bonds fully repaid before 2000 Q1 or issued after 2022 Q4, i.e., we require that bonds are outstanding during our sample period (2000 - 2022).
10. We restrict the sample to fixed coupon bonds [i.e., we exclude bonds with coupon_type field equal to "V" (variable) or "Z" (for zero coupon)].
11. We restrict the sample to bonds with credit rating value from Mergent FISD.

The following table summarise the sample selection:

Table A-3: Sample selection process

Filter	Number of observations: Bond IDs	GVKEYs	Issuer CUSIPs
Total raw from Mergent FISD "ISSUE" table	602,446		21,000
1. Only selected bond types	256,012		15,116
2. Only non-preferred securities by US entities	154,155		10,100
3. In Mergent "Amout_outstanding" table	148,239		9,620
4. Can be linked to Compustat	128,933	3,195	6,603
5. Excluding financials and utilities	34,191	2,168	3,804
6. Information over at least 3 consecutive years	34,042	2,048	3,714
7. Excluding bonds by financial subsidiaries	27,541	2,048	3,668
8. Only bonds outstanding during sample period*	24,190	1,958	3,424
9. Excluding convertibles and exchangeable**	24,108	1,928	3,389
10. Excluding floating rate and zero coupon bonds	21,836	1,902	3,295
11. Excluding bonds without credit rating value	21,508	1,853	3,213
Final sample	21,508	1,853	3,213
thereof rating at issuance IG	16,015	817	1,551
thereof rating at issuance HY	5,493	1,411	1,953
thereof senior unsecured	4,734	1,332	1,754

*2000 Q1 to 2022 Q4.
**A number of 144A bonds in our sample were exchanged for an identical publicly registered bond(s), and are identified in Mergent via the action_type “Issue Exchanged” (“X”). We follow a procedure similar to that described in Powers (2021) in order to remove these original 144A bonds and instead only keep the related publicly registered securities that were issued in their place.

A.2.2 Constructing bond history

This section describes our reconstruction of the history of a corporate bond, from its issuance to its retirement, using data from Mergent. We use both the AMOUNT_OUTSTANDING and the AMT_OUT_HIST tables to construct a historical time series of outstanding balance for each bond issue in Mergent. In order to reconstruct the full history — at monthly frequency — of the life of a given bond, we use the following decomposition, for each bond i in our sample:

$$N_{i,t+1} = N_{i,t} + I_{i,t} - (A_{i,t} + P_{i,t} + D_{i,t}) - U_{i,t}, \tag{A1}$$

where $N_{i,t}$ is the outstanding bond balance at the *beginning* of period t, $A_{i,t}$ represents scheduled amortizations during period t, $D_{i,t}$ represents notional amount defaulted upon during period t, $P_{i,t}$ represents total unscheduled principal repayments during period t, $I_{i,t}$ represents total principal amount issued (either during the original issuance, or via up-sizing) during period t, and $U_{i,t}$ are unexplained changes in the bond notional balance. Those various (flow) quantities are identified from the AMOUNT_OUTSTANDING and the AMT_OUT_HIST tables, by adding up the action_amount field for the relevant period for the relevant bond, using the following classification of the action_type field:

- we label as default $D_{i,t}$ an action_amount with action_type Reorganization ("R");
- we label as scheduled amortization $A_{i,t}$ an action_amount with related action_type Issue Matured ("IM") or Sinking Fund Payment ("S");
- we label as issuance $I_{i,t}$ an action_amount with related action_type Initial Offering of An Issue ("I"), Initial Offering Increase ("II"), Initial Load ("IL"), Over-Allotment ("OA") or Reopening ("RO");
- we label as prepayment $P_{i,t}$ an action_amount with related action_type Balance of Issue Called ("B"), Entire Issue Called ("E"), Issue Repurchased ("IRP"), Optional Sinking Fund Increase ("O"), Part of an Issue Called ("P") or Issue Tendered ("T");
- all action_amount with other action_type are labeled as unexplained activities $U_{i,t}$.

A.2.3 Strike price of call provisions

This section describes (a) how we calculate the strike price for make-whole call provisions, and (b) how we find the strike price for fixed-price call options. For the avoidance of doubt, the strike price is the price at which a firm can contractually call the related bond.

We start with the REDEMPTION table from the Mergent database. This table provides detailed information on how issuers can redeem a bond issue before maturity. For instance, it indicates the frequency at which a bond can be called (e.g., annual, continuously, ...), as well as the start and end dates for make-whole or fixed-price call provisions, if applicable.

Strike price for make-whole call options We first obtain the exact value of the make-whole premium of the bond from the REDEMPTION table. If no value is available (which is only the case for 136 bonds), we assume that the make-whole premium is either a) 25 bps for IG bonds; or b) 50 bps for HY bonds, see Figure A-1.

Next, we create a make-whole call schedule consisting of all call dates for the respective bond. To do this, we take the start of the make-whole call period as the first call date and then find the subsequent call dates based on the call frequency of the bond. We repeat this process until we reach the end of the make-whole call period. For each call date t we then calculate the make-whole call price for bond i as follows:

$$P_{i,t}^{make-whole\ call} = \sum_{n=1}^{N} \frac{CF_{i,t+n}}{(1 + swap\_rate_{i,t+n} + make\_whole\_premium_i)^n},$$

where $CF_{i,t+n}$ is the future cash flow to bond holders at time $t+n$, $make\_whole\_premium_i$ is the bond-specific make-whole premium, and $swap\_rate_{i,t+n}$ is the swap rate at time $t+n$. $t+N$ is either the maturity date for pure make-whole callable bonds or the first fixed-price call date for hybrid bonds. Specifically, for pure make-whole callable bonds, the future cash flows in the above calculation include all future coupons and the final principle payment at maturity date. For hybrid callable bonds, the future cash-flows only include all future coupons until the first fixed-price call date, as well as the first fixed-price call price, which we find through the procedure described in the next section.

Strike price for fixed-price call options We utilize the CALL_SCHEDULE table and ANNOUNCED_CALL table from the Mergent database. Combining these tables, we obtain a complete list of fixed-price call dates and fixed-price call prices for each bond that is fixed-price callable at any point during its life.

We carefully check the call frequency for each bond using the REDEMPTION table. If there are any missing dates in the fixed-price call schedule, we carry the last available call price forward, assuming the strike price stays the same.

A.3 Theoretical Appendix

A.3.1 The simple setting

A.3.1.1 Equity Valuation and Optimal Refinancing Strategy

Let Θ be the 2×2 matrix corresponding to the generator of the discrete state, continuous time, Markov process s_t. We postulate that the optimal strategy is a stopping time of the form $\tau_r = \inf\{t \geq 0 : T_t \leq T^*, s_t = 1\}$, i.e. the firm prepays as soon as the time-to-maturity T_t falls below T^* and capital market conditions are good. For $m \geq T \geq T^*$, the equity value solves the following Hamilton-Jacobi-Bellman ("HJB") equation:

$$(r + \lambda + \theta_u)E_0(T) = y - cb - \partial_T E_0(T) + \theta_u E_1(T)$$
$$(r + \lambda + \theta_d)E_1(T) = y - cb - \partial_T E_1(T) + \theta_d E_0(T)$$

Let us use vector notation: $\vec{E}(T)$ denotes the equity value vector $(E_0(T), E_1(T))'$, and $\vec{D}(T)$ denotes the corresponding debt price vector. The system of ODEs above can be written in vector form

$$\partial_T \vec{E} = -M\vec{E} + (y - cb)\,\vec{1},$$

where we have introduced the 2×2 matrix $M := (r + \lambda)I - \Theta$. We note $e_0 := \frac{y-cb}{r+\lambda}$ the firm equity value absent roll-over risk. This above system of differential equations can be solved, for given final condition $\vec{E}(m)$:

$$\vec{E}(T) = \exp\left(M(m - T)\right)\left(\vec{E}(m) - e_0\vec{1}\right) + e_0\vec{1}$$

For $T \leq T^*$, under our postulated equilibrium, the firm prepays its outstanding debt and refinances when in the good capital market state; instead, in the bad capital market state, the firm waits for a state transition:

$$(r + \lambda + \theta_u)E_0(T) = y - cb - \partial_T E_0(T) + \theta_u\left(E_1(m) - b\kappa(T)\right)$$
$$E_1(T) = E_1(m) - b\kappa(T)$$

At maturity $T = 0$, if capital market conditions are good, the firm rolls over its debt without any call premium (since $\kappa(0) = 0$), and the system is "reset"; instead, if capital market conditions are bad, the firm cannot roll over its debt and is forced to default. We thus have the boundary conditions

$$E_0(0) = 0 \qquad E_1(0) = E_1(m)$$

Note in particular that for $T < T^*$, E_1 is decreasing in T. We can also integrate the differential equation for E_0, for $T \leq T^*$ and obtain:

$$E_0(T) = \int_0^T e^{-(r+\lambda+\theta_u)(T-t)}\left[y - cb + \theta_u\left(E_1(m) - b\kappa(t)\right)\right]dt$$

We have thus determined the equity values E_0, E_1 on the entire state space, up to 2 constants $E_0(m), E_1(m)$. Those constants are determined by the condition that $\vec{E}$ be continuous at $T = T^*$. This yields the system of equations

$$e^{M(m-T^*)}\left(\vec{E}(m) - e_0\vec{1}\right) + e_0\vec{1} = N(T^*)\vec{E}(m) + \vec{g}(T^*),$$

where we have introduced

$$N(T^*) := \begin{pmatrix} 0 & \theta_u \int_0^{T^*} e^{-(r+\lambda+\theta_u)(T^*-t)} dt \\ 0 & 1 \end{pmatrix} \quad \vec{g}(T^*) := \begin{pmatrix} \int_0^{T^*} e^{-(r+\lambda+\theta_u)(T^*-t)} \left[y - cb - \theta_u b\kappa(t)\right] dt \\ -b\kappa(T^*) \end{pmatrix}$$

The solution to this equation is

$$\vec{E}(m) = \left(e^{M(m-T^*)} - N(T^*)\right)^{-1} \left[\vec{g}(T^*) + e_0\left(e^{M(m-T^*)} - I\right)\vec{1}\right]$$

Note that since E_1 is continuous at $T = T^*$, it must be the case that E_0 is continuously differentiable at that point. The optimal refinancing time is the time that maximizes the equity value. It is also the unique time such that the function E_1 is continuously differentiable at $T = T^*$. Indeed, notice that the maximization of $\vec{E}(m)$ w.r.t. T^* leads to the first order condition

$$\vec{g}'(T^*) - e_0 M e^{M(m-T^*)}\vec{1} + \left(M e^{M(m-T^*)} + N'(T^*)\right)\vec{E}(m) = 0$$

The second row of $N(T^*)$ is independent of T^*, which means that the second row of $N'(T^*)$ only consists of zeros. Thus, the second row of the first order condition w.r.t. T^* can be re-written

$$-(0 \quad 1)' M e^{M(m-T^*)}\left(\vec{E}(m) - e_0\vec{1}\right) = -b\kappa'(T^*)$$

This expression exactly corresponds to $\partial_T E_1(T^*_-) = \partial_T E_1(T^*_+)$ — i.e. that E_1 is continuously differentiable at $T = T^*$. This expression also has another interpretation; the left-hand side is the equity marginal value of time-to-maturity $\partial_T E_1$, whereas the right-hand side is the marginal prepayment cost $-\partial_T(b\kappa(T))$. Naturally, at the optimal prepayment time-to-maturity T^*, the firm equalizes marginal cost to marginal benefit.

A.3.1.2 Debt valuation

$D_s(T)$ is the price per unit of debt in state s, with time-to-maturity equal to T years. When the firm defaults, creditors recover $\rho \in (0,1)$. D_s solves, for $m \geq T \geq T^*$:

$$(r+\lambda+\theta_u)D_0(T) = c + \lambda\rho - \partial_T D_0(T) + \theta_u D_1(T)$$
$$(r+\lambda+\theta_d)D_1(T) = c + \lambda\rho - \partial_T D_1(T) + \theta_d D_0(T)$$

With $d_0 := \frac{c+\lambda\rho}{r+\lambda}$ the bond price absent roll-over risk, the solution to these equations is

$$\vec{D}(T) = \exp\left(M(m-T)\right)\left(\vec{D}(m) - d_0\vec{1}\right) + d_0\vec{1}$$

For $T \leq T^*$, we then have

$$(r+\lambda+\theta_u)D_0(T) = c + \lambda\rho - \partial_T D_0(T) + \theta_u\left(D_1(m) + \kappa(T)\right)$$
$$D_1(T) = D_1(m) + \kappa(T)$$

Finally, we have the boundary conditions

$$D_0(0) = \rho \qquad D_1(0) = D_1(m)$$

We can integrate the differential equation for D_0 to obtain, for $T \leq T^*$:

$$D_0(T) = \int_0^T e^{-(r+\lambda+\theta_u)(T-t)}\left(c + \lambda\rho + \theta_u\left(D_1(m) + \kappa(t)\right)\right)dt + \rho e^{-(r+\lambda+\theta_u)T}$$

We have thus determined the debt prices D_0, D_1 on the entire state space, up to two constants $D_0(m), D_1(m)$. Those 2 constants are pinned down by the continuity of $\vec{D}$ at $T = T^*$. This system can be re-written

$$e^{M(m-T^*)}\left(\vec{D}(m) - d_0\vec{1}\right) + d_0\vec{1} = N(T^*)\vec{D}(m) + \vec{h}(T^*),$$

where we have introduced

$$\vec{h}(T^*) := \begin{pmatrix} \int_0^{T^*} e^{-(r+\lambda+\theta_u)(T^*-t)}\left(c + \lambda\rho + \theta_u\kappa(t)\right)dt + \rho e^{-(r+\lambda+\theta_u)T^*} \\ \kappa(T^*) \end{pmatrix}$$

The solution to this equation is

$$\vec{D}(m) = \left(e^{M(m-T^*)} - N(T^*)\right)^{-1} \left[\vec{h}(T^*) + d_0 \left(e^{M(m-T^*)} - I\right) \vec{1}\right]$$

The coupon c is then pinned down by the condition that $D_1(m) = 1$.

A.3.1.3 Other equilibrium objects

The credit spread of the bond $x_s(T)$ is defined, for any equilibrium, via

$$D_s(T) = \int_0^T e^{-(r+x_s(T))t} c dt + e^{-(r+x_s(T))T} = \frac{c}{r + x_s(T)} \left[1 - e^{-(r+x_s(T))T}\right] + e^{-(r+x_s(T))T}$$

We can then focus on the non-callable debt equilibrium — i.e. the environment in which firms face rollover risk but only issue non-callable debt. With the risk of a credit market shut-down and non-callable debt, the par coupon c needs to satisfy $D_{nc,1}(m) = 1$, where $\vec{D}_{nc}$ solves, for $T \leq m$:

$$\begin{aligned} \partial_T \vec{D}_{nc} &= (c + \lambda\rho)\vec{1} - M\vec{D}_{nc} \\ \vec{D}_{nc}(0) &= \begin{pmatrix} \rho \\ 1 \end{pmatrix} \end{aligned}$$

Thus, the non-callable debt price satisfies

$$\vec{D}_{nc}(T) = \exp\left(-MT\right) \begin{pmatrix} \rho - d_{nc} \\ 1 - d_{nc} \end{pmatrix} + \begin{pmatrix} d_{nc} \\ d_{nc} \end{pmatrix},$$

where the constant $d_{nc} = \frac{c+\lambda\rho}{r+\lambda}$. The equity value vector $\vec{E}_{nc}$ then satisfies

$$\vec{E}_{nc}(T) = \exp\left(M(m-T)\right) \left(\vec{E}_{nc}(m) - e_{nc}\vec{1}\right) + e_{nc}\vec{1},$$

with $e_{nc} = \frac{y-cb}{r+\lambda}$. We pin down $\vec{E}_{nc}(m)$ via $E_{nc,0}(0) = 0$ and $E_{nc,1}(0) = E_{nc,1}(m)$.

Internet Appendix to "The Art of Timing: Managing Sudden Stop Risk in Corporate Credit Markets" (not intended for publication)

Lin Ma, Daniel Streitz, and Fabrice Tourre

IA.1 Linking bonds to issuer GVKeys

In this section, we describe how we assign Compustat GVKeys to bonds in the Mergent dataset.

We first leverage the Bond-CRSP Link table from WRDS to identify issuer GVKeys. The link table gives a dynamic match between CUSIP and PERMCO (i.e., captures dynamic ownership changes), which can be then translated to GVKeys using information from CRSP Compustat Merged (CCM). We first inner join our bond-month panel described in Section A.2.2 with the Bond-CRSP link table using (a) the first 6-digits of the bond's CUSIP and (b) the month and year of the bond outstanding period. We then link PERMCOs to CCM to obtain issuer GVKeys. This step allows us to find GVkeys for around [80%] of bonds in our final sample.

For the remaining bonds for which we cannot identify the issuer in Compustat with the procedure described above, we proceed as follows: we only utilize information as of bond issuance (i.e., we assume the bond-firm relationship is fixed over the life of the bond) and match the bond-level data directly to the CCM database, using (a) the 6-digit CUSIP and (b) the month and year when the bond was issued (according to Mergent's 'ISSUE' table). This approach allows us to determine the issuer GVKey for about [14]% of all bonds in our final sample.

Finally, we use the following 3-step approach that is responsible for the remaining [6%] of the matches:

1. We leverage the issuer_id field available in the 'ISSUER' table in Mergent. For each bond in Mergent that does not yet have an issuer GVKey assigned to it, we check its issuer_id, and see if the id has already been assigned to an issuer GVKey in any of the steps described above (i.e., we check if other bonds issued by the same issuer_id have been matched with an issuer GVKey); if this is not the case, we proceed to the next step;

2. We leverage the agent_id field avaible in the 'ISSUER' table in Mergent. Different issuer_ids can be connected through common agent_ids (e.g., Delta Airline Through Trust and Delta Airlines have different issuer_ids but identical agent_ids). We then repeat step 1.), i.e., we check if GVKeys can be obtained from issuer_ids that share the same agent_id. If this is not the case, we proceed to the next and final step;

3. We leverage the parent_id field avaible in the 'ISSUER' table. This allows us to identify the parent GVKey for bonds issued by subsidiaries (e.g., bond issued by Fox Kids worldwide Inc can be linked to Disney (Walt) Co by using this approach.).

IA.2 Model: the general setting

IA.2.1 Model solution

Let us denote $\kappa_{ij}(T) := \kappa(s_i, T, \mathcal{C}(j))$. Let $\mathcal{L}$ be the linear operator mapping continuously differentiable vector-valued functions into vector-valued functions, and defined, for a vector-valued function $\vec{h}(T) = (h_1(T), ..., h_n(T))'$, via

$$\left(\mathcal{L}\vec{h}\right)_i (T) := \sum_{\ell=1}^{n} \theta_{i\ell} h_\ell(T) - \partial_T h_i(T)$$

$E_{ij}(T)$ must satisfy the variational inequality, for all $i, j \leq n$ and $T \in [0, m]$:

$$\max \Big[-(r_i + \lambda_i) E_{ij}(T) + y_i - \mathcal{C}(j) b + \left(\mathcal{L}\vec{E}_{\cdot j}\right)_i (T); \\ \pi_i \left(E_{ii}(m) - \kappa_{ij}(T) b\right) - E_{ij}(T) \Big] = 0, \quad \text{(IA1)}$$

where we have noted $\vec{E}_{\cdot j} := \left(E_{1j}, ..., E_{nj}\right)'$. The first term of the max operator in (IA1) is the HJB equation, encoding the evolution of the equity value as the state s_t transitions (according to the generator Θ), as default occurs (at intensity λ_i), and as time passes. The second term encodes the option for shareholders to refinance: at all time, their equity value $E_{ij}(T)$ must weakly dominate the value of exercising the call, with payoff $\pi_i \left(E_{ii}(m) - \kappa_{ij}(T) b\right)$. $E_{ij}(T)$ must also satisfy the boundary conditions, for all $i, j \leq n$:

$$E_{ij}(0) = \pi_i E_{ii}(m) \quad \text{(IA2)}$$

On the debt maturity date $T = 0$, (IA2) encodes the roll-over risk taken by firms given our imperfect credit markets assumption. Let T^*_{ij} the optimal prepayment time-to-maturity for a firm with coupon $\mathcal{C}(j)$ in state i:

$$T^*_{ij} = \sup\{T \in [0, m] : E_{ij}(T) = \pi_i \left(E_{ii}(m) - \kappa_{ij}(T) b\right)\}$$

The optimal prepayment times-to-maturity must satisfy, for all $1 \leq i, j \leq n$

$$\partial_T E_{ij}(T^*_{ij}) = -b \partial_T \kappa_{ij}(T^*_{ij}).$$

They also satisfy

$$T > T^*_{ij} : \pi_i \left(E_{ii}(m) - \kappa_{ij}(T) b \right) < E_{ij}(T)$$
$$T \leq T^*_{ij} : \pi_i \left(E_{ii}(m) - \kappa_{ij}(T) b \right) = E_{ij}(T)$$

The bond price $D_{ij}(T)$ satisfies the following asset pricing equation, for $T \geq T^*_{ij}$:

$$(r_i + \lambda_i) D_{ij}(T) = \mathcal{C}(j) + \sum_{\ell : T \geq T^*_{\ell j}} \theta_{i\ell} D_{\ell j}(T) + \sum_{\ell : T < T^*_{\ell j}} \theta_{i\ell} \left[\pi_\ell \left(1 + \kappa_{\ell j}(T) \right) + (1 - \pi_\ell) \rho_\ell \right] + \lambda_i \rho_i - \partial_T D_{ij}(T) \quad \text{(IA3)}$$

(IA3) insures that the discounted debt price is a martingale; it takes into account the passage of time, jumps to default at intensity λ_i, and state transitions according to Θ, which can lead either (a) to a price change, or (b) to the exercise of a call option by shareholders, with some likelihood of a successful refinancing, but also some likelihood of a failed refinancing and ultimate default. The bond price $D_{ij}(T)$ must also satisfy the following terminal conditions:

$$D_{ij}(T) = \pi_i \left(1 + \kappa_{ij}(T) \right) + (1 - \pi_i) \rho_i, \qquad T < T^*_{ij} \quad \text{(IA4)}$$
$$D_{ij}(0) = \pi_i + (1 - \pi_i) \rho_i \quad \text{(IA5)}$$

(IA4) encodes the payoff for creditors in states where the firm finds it optimal to immediately refinance. Finally, on the bond maturity date $T = 0$, (IA5) encodes the fact that firms are either able to access credit markets and refinance (with probability π_i) or are shut down from credit markets and default (with probability $1 - \pi_i$). The coupon $\mathcal{C}(j)$ when refinancing occurs in state j must satisfy $D_{jj}(m) = 1$ for all $j \leq n$.

IA.2.2 Stationary distribution

Under the assumption that under the statistical measures, defaults do not occur (or under the assumption that once a firm defaults, another firm is created, with identical coupon and bond time-to-maturity as the firm that just defaulted), one can define a long-run distribution $f_{ij}(T)$ of firms in aggregate state i, with coupon $\mathcal{C}(j)$, with time-to-maturity on the interval $[T, T + dT]$. This long-run distribution is defined via:

$$f_{ij}(T) dT := \lim_{t \to +\infty} \frac{1}{t} \int_0^t 1_{\{s_u = i, T_u \in [T, T+dT], c_u = \mathcal{C}(j)\}} du$$

For $T < T^*_{ij}$, this long-run distribution satisfies of course $f_{ij}(T) = 0$. For $T \geq T^*_{ij}$, it satisfies the Kolmogorov-Forward equation

$$0 = \left(\mathcal{L}^* \vec{f}_{\cdot j}\right)_i (T),$$

where $\mathcal{L}^*$ is the adjoint of the operator $\mathcal{L}$, which, in our case, takes the following form:

$$\left(\mathcal{L}^* \vec{f}_{\cdot j}\right)_i (T) := \sum_{\ell=1}^{n} \theta_{\ell i} f_{\ell j}(T) + \partial_T f_{ij}(T)$$

Finally, the distribution $f_{ij}(T)$, at the point where bonds are newly issued (i.e. when $T = m$), must satisfy:

$$\begin{aligned} f_{ij}(m) &= 0, i \neq j \\ f_{ii}(m) &= \sum_{j=1}^{n} \left[f_{ij}(T^*_{ij}) + \sum_{\ell : T^*_{ij} > T^*_{\ell j}} \theta_{\ell i} \int_{T^*_{\ell j}}^{T^*_{ij}} f_{\ell j}(T) dT \right] \end{aligned}$$

This latter equation insures that the flow of firms issuing debt in state i is equal to the flow of firms refinancing. One can then use our long-run distribution to compute the ergodic average prepayment rate. Such rate is equal to:

$$\lim_{t \to +\infty} \frac{N_t}{t} = \sum_{i=1}^{n} \sum_{j=1}^{n} \left[f_{ij}\left(T^*_{ij}\right) + \sum_{\ell : T^*_{ij} < T^*_{\ell j}} \theta_{i\ell} \int_{T^*_{ij}}^{T^*_{\ell j}} f_{ij}(T) dT \right]$$

This prepayment rate is the sum of two terms. The first term arises with the passage of time, as firms in state i with coupon $\mathcal{C}(j)$ end up prepaying when their time-to-maturity eventually falls below T^*_{ij}; the second term arises with state transitions from state i to state ℓ, for instance as interest rates suddenly decline from $r(s_i)$ to $r(s_\ell) < r(s_i)$, and the optimal prepayment time-to-maturity in the new state ℓ is greater than current time-to-maturity T, i.e. when $T^*_{\ell j} > T > T^*_{ij}$. Similarly, one can compute ergodic average call premium payments, as well as ergodic average coupon payments, as follows:

$$\lim_{t \to +\infty} \frac{1}{t} \int_0^t \kappa\left(s_t, T_t, c_t\right) dN_t = \sum_{i=1}^{n} \sum_{j=1}^{n} \left[f_{ij}\left(T^*_{ij}\right) \kappa_{ij}\left(T^*_{ij}\right) + \sum_{\ell : T^*_{ij} < T^*_{\ell j}} \theta_{i\ell} \int_{T^*_{ij}}^{T^*_{\ell j}} f_{ij}(T) \kappa_{ij}(T) dT \right]$$

$$\lim_{t \to +\infty} \frac{1}{t} \int_0^t c_t dt = \sum_{1 \le i,j \le n} \int_0^{T_{ij}^*} f_{ij}(T) \, \mathcal{C}(j) dT,$$

IA.2.3 "Standard" credit spreads

Let $x_{ij}(T)$ be the credit spread of a bond with time-to-maturity T, conditional on the current aggregate state being $s_t = i$ and conditional on the coupon being equal to c_j; $x_{ij}(T)$ solves

$$D_{ij}(T) = \mathbb{E}_{i,j,T} \left[\int_0^T e^{-\int_0^t (r(s_u)+x_{ij}(T))du} c_j dt + e^{-\int_0^T (r(s_u)+x_{ij}(T))du} \right]$$

Note that the integrals inside the expectation are taken from time 0 until contractual time-to-maturity T. In other words, this "naive" credit spread calculation does not factor in the fact that the bond *effective life* might be shorter than T years. In vector form, introducing the matrix $N(\vec{x}) := \text{diag}(r + x) - \Theta$, the unknown vector $\vec{x}_{\mathcal{E}_j,j}(T)$ solves

$$\exp\left(N_{\mathcal{E}_j}(\vec{x})T\right) \vec{D}_{\mathcal{E}_j,j}(T) - \vec{1}_{\mathcal{E}_j} = c_j N_{\mathcal{E}_j}(\vec{x})^{-1} \left[\exp\left(N_{\mathcal{E}_j}(\vec{x})T\right) - I\right] \vec{1}_{\mathcal{E}_j}$$

IA.2.4 "Option-adjusted" credit spreads

Let $y_{ij}(T)$ be the option adjusted credit spread of the bond, conditional on the current aggregate state being i, the bond coupon being equal to c_j and the time-to-maturity being equal to T; $y_{ij}(T)$ solves

$$D_{ij}(T) = \mathbb{E}_{i,j,T} \left[\int_0^{T \wedge \tau_r} e^{-\int_0^t (r(s_u)+y_{ij}(T))du} c_j dt + e^{-\int_0^{T \wedge \tau_r} (r(s_u)+y_{ij}(T))du} \right]$$

Note now that the integrals inside the expectation are taken from time 0 until the smaller of (a) the maturity T, or (b) the (random) prepayment time τ_r. In other words, this "option adjusted" credit spread calculation takes into account the fact that the bond *effective life* might be shorter than T years.

IA.2.5 Term structure of Interest rates

In this model, when the aggregate state is s, a risk-free bond with coupon c and T time-to-maturity has a price $P_s(T)$, which can be computed via

$$(\mathrm{diag}(r_i) - \Theta)\,\vec{P} = c\vec{1} - \partial_T \vec{P} \qquad\qquad \vec{P}(0) = \vec{1}$$

Introduce the matrix $N(\varsigma) := \mathrm{diag}(r_i + \varsigma) - \Theta$, then we have

$$\vec{P}(T) = \exp\left(-N(0)T\right)\left(I - cN(0)^{-1}\right)\vec{1} + cN(0)^{-1}\vec{1}$$

The par coupon for a non-callable bond in state i then satisfies $P_i(m) = 1$:

$$c_i = \frac{1 - \left[\exp\left(-N(0)m\right)\vec{1}\right]_i}{\left[\left(I - \exp\left(-N(0)m\right)\right)N(0)^{-1}\vec{1}\right]_i}$$

IA.2.6 Vanilla defaultable debt

In a model where the firm does not have the option to prepay and there is no roll-over risk, in state s, a defaultable bond with time-to-maturity of T years and coupon c has a price $P_s(T)$, which can be computed via

$$(\mathrm{diag}(r_i + \lambda_i) - \Theta)\,\vec{P}(T) = c\vec{1} + \vec{\rho} \odot \vec{\lambda} - \partial_T \vec{P}(T) \qquad\qquad \vec{P}(0) = \vec{1}$$

With the matrix $M := \mathrm{diag}(r_i + \lambda_i) - \Theta$, we have

$$\vec{P}(T) = \exp\left(-MT\right)\vec{1} + \left(I - \exp\left(-MT\right)\right)M^{-1}\left[c\vec{1} + \vec{\rho} \odot \vec{\lambda}\right]$$

The par coupon for a non-callable defaultable bond in state i then satisfies $P_i(m) = 1$:

$$c_i = \frac{1 - \left[\exp\left(-Mm\right)\vec{1} + \left(I - \exp\left(-Mm\right)\right)M^{-1}\vec{\rho} \odot \vec{\lambda}\right]_i}{\left[\left(I - \exp\left(-Mm\right)\right)M^{-1}\vec{1}\right]_i}$$

IA.2.7 Make-whole call prices for pure make-whole callable bonds

In this model, in aggregate state s, the make-whole call price for a bond with time-to-maturity T years, coupon c and makewhole spread ς is $P_s(T,\varsigma)$, which can be computed via

$$(\mathrm{diag}(r_i+\varsigma)-\Theta)\,\vec{P}(T)=c\vec{1}-\partial_T\vec{P}(T) \qquad \vec{P}(0)=\vec{1}$$

In other words:

$$\vec{P}(T,\varsigma)=\exp\left(-N(\varsigma)T\right)\left(I-cN(\varsigma)^{-1}\right)\vec{1}+cN(\varsigma)^{-1}\vec{1}$$

IA.2.8 Make-whole call prices for hybrid callable bonds

In this model, in aggregate state s, the make-whole call price for a hybrid bond with time-to-maturity T years, a coupon c, and makewhole spread ς, when the time-to-maturity at which the bond becomes fixed-call callable is $T_f < m$ and the first fixed-call price is P_f is $P_s(T,\varsigma,T_f,P_f)$, which can be computed via

$$(\mathrm{diag}(r_i+\varsigma)-\Theta)\,\vec{P}(T)=c\vec{1}-\partial_T\vec{P}(T) \qquad \vec{P}(T_f)=P_f\vec{1}$$

In other words, for $T \geq T_f$:

$$\vec{P}(T,\varsigma)=\exp\left(-N(\varsigma)(T-T_f)\right)\left(P_f I-cN(\varsigma)^{-1}\right)\vec{1}+cN(\varsigma)^{-1}\vec{1}$$

IA.2.9 Interest rates on non-callable debt

Assume now that we want to compute the par coupon that a firm would need to pay in order to issue m-maturity non-callable debt, *keeping the equilibrium with callable debt fixed*. When that bond has time-to-maturity t, when the time-to-maturity of the firm's callable debt is T, when the current firm's debt coupon is $\mathcal{C}(j)$ and the current aggregate state is $s_t=i$, the price of a non-callable instrument with coupon rate c is

$$\mathbb{E}_{i,j,T}\left[\int_0^{\tau_d\wedge t} e^{-\int_0^v r(s_u)du}c\,dv+e^{-\int_0^{\tau_d\wedge t} r(s_u)du}\left[1_{\{\tau_d\leq t\}}\rho_{\tau_d}+1_{\{t\leq\tau_d\}}\right]\right],$$

Introduce the following objects:

$$K_{ij}(T,t) := \mathbb{E}_{i,j,T}\left[\int_0^{\tau_d\wedge t} e^{-\int_0^v r(s_u)du} dv\right]$$
$$R_{ij}(T,t) := \mathbb{E}_{i,j,T}\left[e^{-\int_0^{\tau_d\wedge t} r(s_u)du} 1_{\{\tau_d\leq t\}}\rho_{\tau_d}\right]$$
$$P_{ij}(T,t) := \mathbb{E}_{i,j,T}\left[e^{-\int_0^{\tau_d\wedge t} r(s_u)du} 1_{\{t\leq \tau_d\}}\right],$$

then the par coupon c_j for non-callable, m-maturity debt issued in state j is

$$c_j = \frac{1-\left(R_{jj}(m,m)+P_{jj}(m,m)\right)}{K_{jj}(m,m)}$$

Then note that we have, for $X = K, R, P$ and $T \geq T^*_{ij}$

$$\begin{aligned}(r_i+\lambda_i)\, X_{ij}(T,t) = F_X(i) &+ \sum_{\ell: T\geq T^*_{\ell j}} \theta_{i\ell} X_{\ell j}(T,t) \\ &+ \sum_{\ell: T< T^*_{\ell j}} \theta_{i\ell}\left[\pi_\ell X_{ii}(m,t)+(1-\pi_\ell)X_{\ell j}(T,t)\right] - \partial_T X_{ij}(T,t) - \partial_t X_{ij}(T,t),\end{aligned}$$

with $F_R(i) = \lambda_i\rho_i, F_K(i) = 1$ and $F_P(i) = 0$, and initial conditions $K_{ij}(T,0) = R_{ij}(T,0) = 0$, and $P_{ij}(T,0) = 1$.

One can then contrast the par coupon payable on non-callable debt in the equilibrium with callable debt with the counterfactual par coupon that would be payable on non-callable debt in the *alternative equilibrium in which the firm was not able to issue callable debt*. In that alternative equilibrium, the market value of t-maturity non-callable debt with coupon c is simply equal to

$$\mathbb{E}_{i,j,T}\left[\int_0^t e^{-\int_0^v (r(s_u)+\lambda(s_u))du}\left(c+\lambda(s_v)\rho(s_v)\right)dv + e^{-\int_0^t (r(s_u)+\lambda(s_u))du}\left[\pi(s_t)+\left(1-\pi(s_t)\right)\rho(s_t)\right]\right].$$

Using this formulation, it is then straightforward to compute the par coupon that would be prevalent in an environment where the firm was not allowed to issue callable debt.

IA.3 Additional figures and tables

Table IA-1: Prepayment size

This table shows statistics about prepayment size (% of initial offering amount) by bond type and prepayment method. # indicates the underlying number of prepayment events within each category.

	Call				Tender				Buyback			
	Mean	Med	p75	#	Mean	Med	p75	#	Mean	Med	p75	#
Non-callable	37.1%	20.4%	77.5%	14	40.0%	35.5%	78.1%	78	43.1%	49.9%	70.3%	20
Fixed price	69.1%	99.2%	100.0%	1,622	78.2%	94.9%	100.0%	479	23.9%	15.0%	31.7%	152
Hybrid (non-MW period)	76.7%	100.0%	100.0%	1,173	72.8%	86.8%	100.0%	320	19.0%	14.5%	27.5%	62
Hybrid (MW period)	69.2%	100.0%	100.0%	485	67.9%	83.3%	100.0%	346	18.4%	11.7%	26.8%	174
Make whole	68.2%	97.7%	100.0%	198	65.8%	73.6%	95.3%	149	21.6%	15.9%	28.7%	63

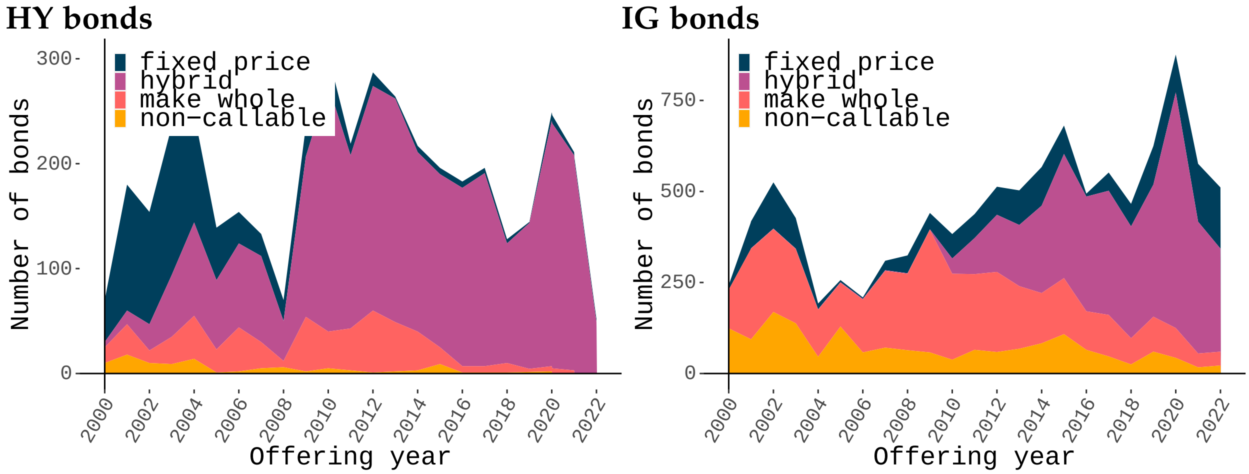


Figure IA-1: Bond call features over time
This figure plots the annual aggregate number of newly issued HY bonds (left) and IG bonds (right) over time. The figure distinguishes between "fixed price" callable bonds, “make-whole” callable bonds, "non-callable" bonds, and "hybrid" callable bonds.

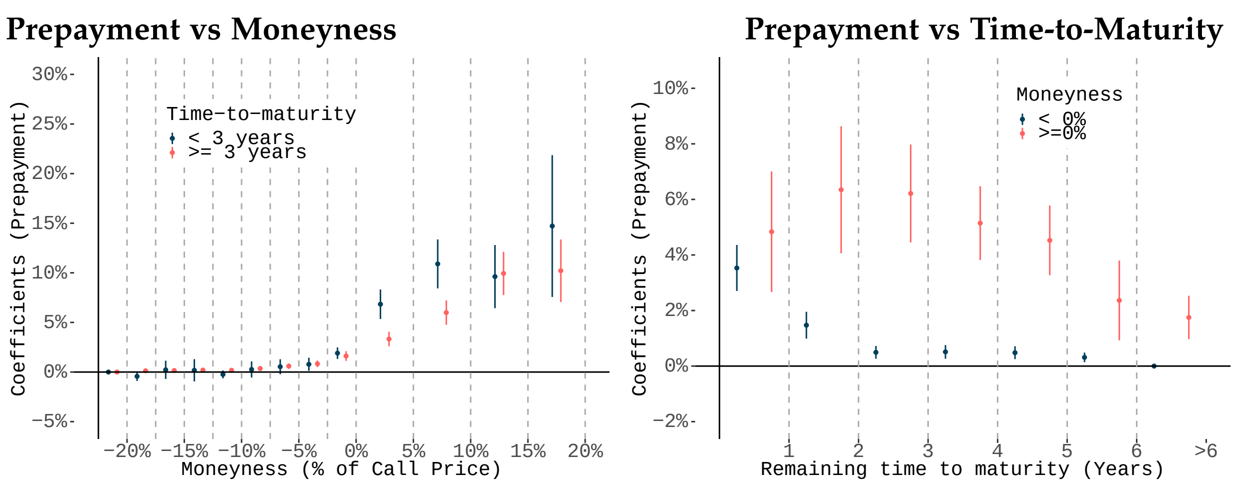


Figure IA-2: Time-to-maturity and moneyness
The left figure shows regression coefficients of an estimation of bond prepayment probabilities by option moneyness bin, after control for the near-to-maturity (< 3 years) dummy of the bond as well as the interaction between those. The right figure shows regression coefficients of an estimation of bond prepayment probabilities by time-to-maturity bin, after control for the in-the-money dummy of the bond as well as the interaction of those two. Firm fixed effect applied for those regressions.

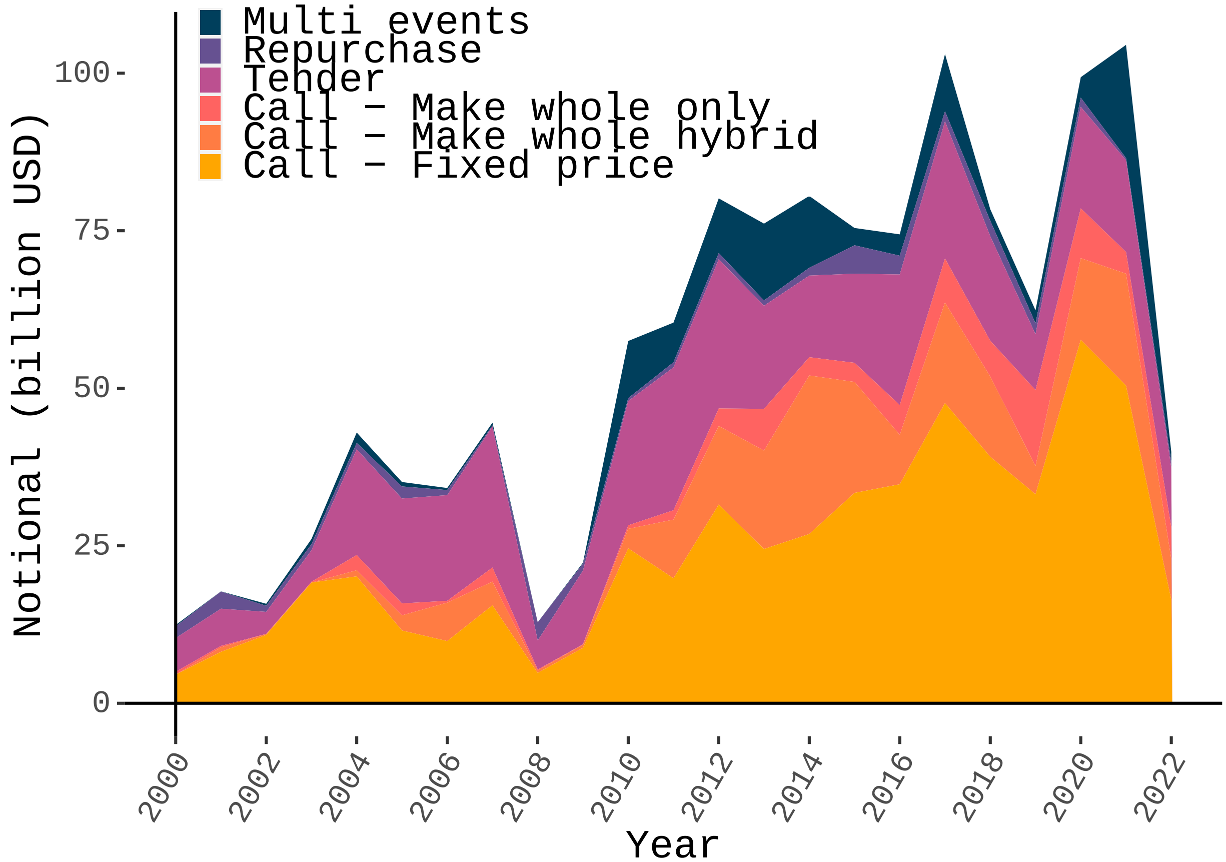


Figure IA-3: Total volume prepaid by action type
The figure displays the aggregate amount of different prepayment events for HY bonds in each year from 2000 to 2019. The figure distinguishes between "fixed price" call provision and "make-whole" call provision. Multiple prepayment events can happen within the same month for a single instrument.

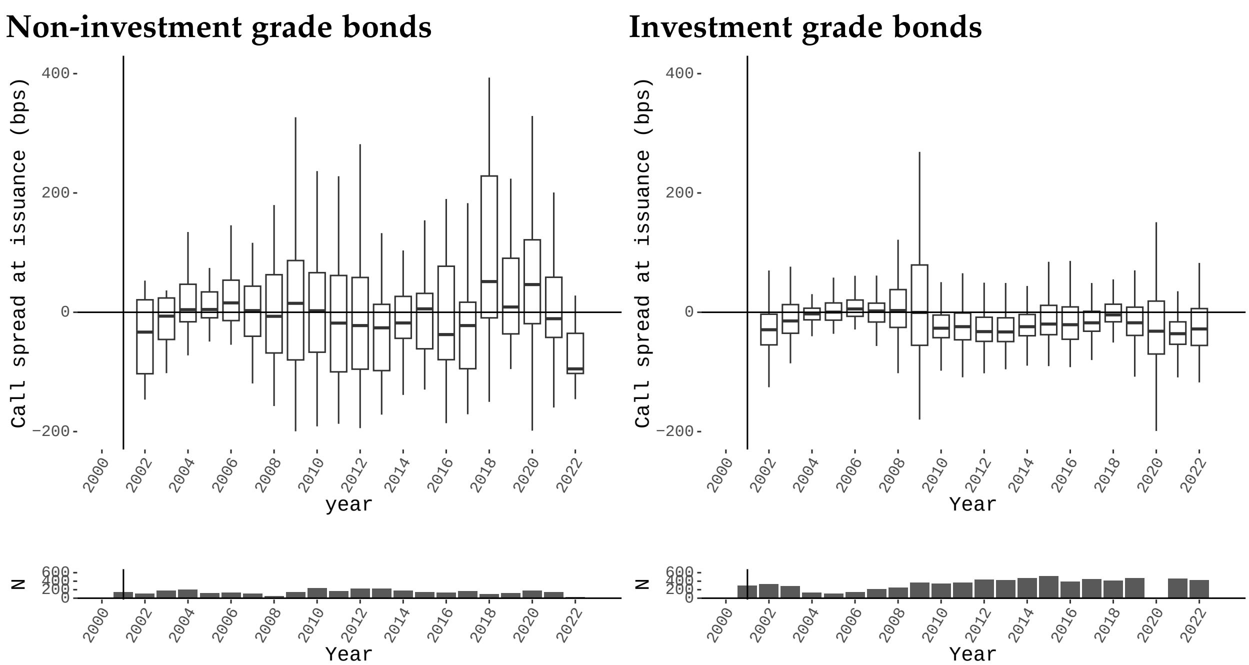
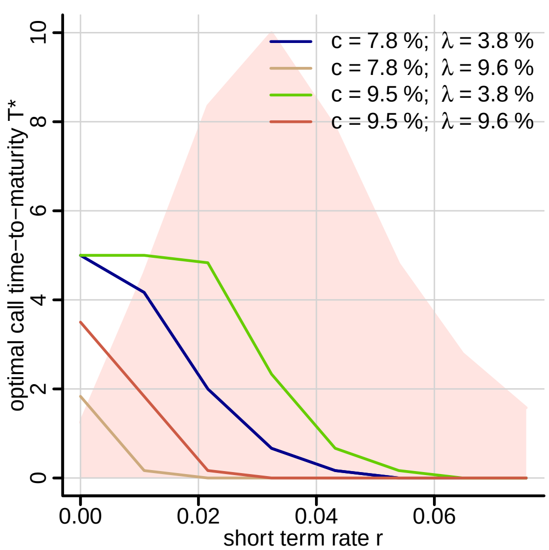


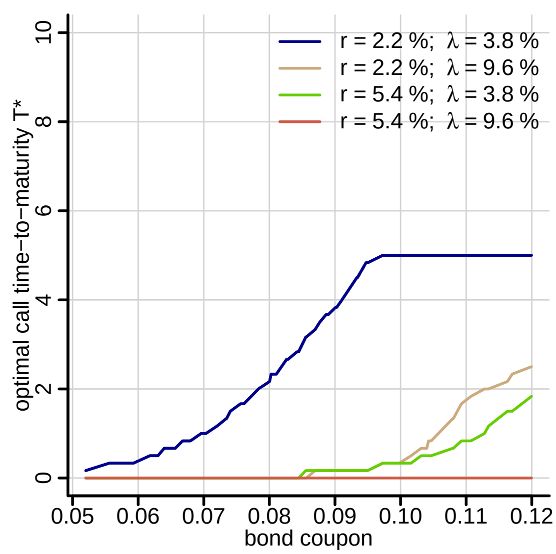
Figure IA-4: Bond call spread at issuance
This figure plots the difference between (a) a callable bond's coupon and (b) the corresponding hypothetical non-callable bond's coupon at the time of its issuance using the synthetic non-callable replication framework of Section 5.2.2 for newly issued HY bonds (left) and IG bonds (right) over time.

Figure IA-5: Optimal call policies

(a): T^* vs. short rates



(b): T^* vs. coupon rates



Left (resp. right) plot shows optimal call time-to-maturity T^* as a function of the short rate r_t (resp. the bond coupon c_t) for various levels of default intensities rates and coupons (resp. short rates).

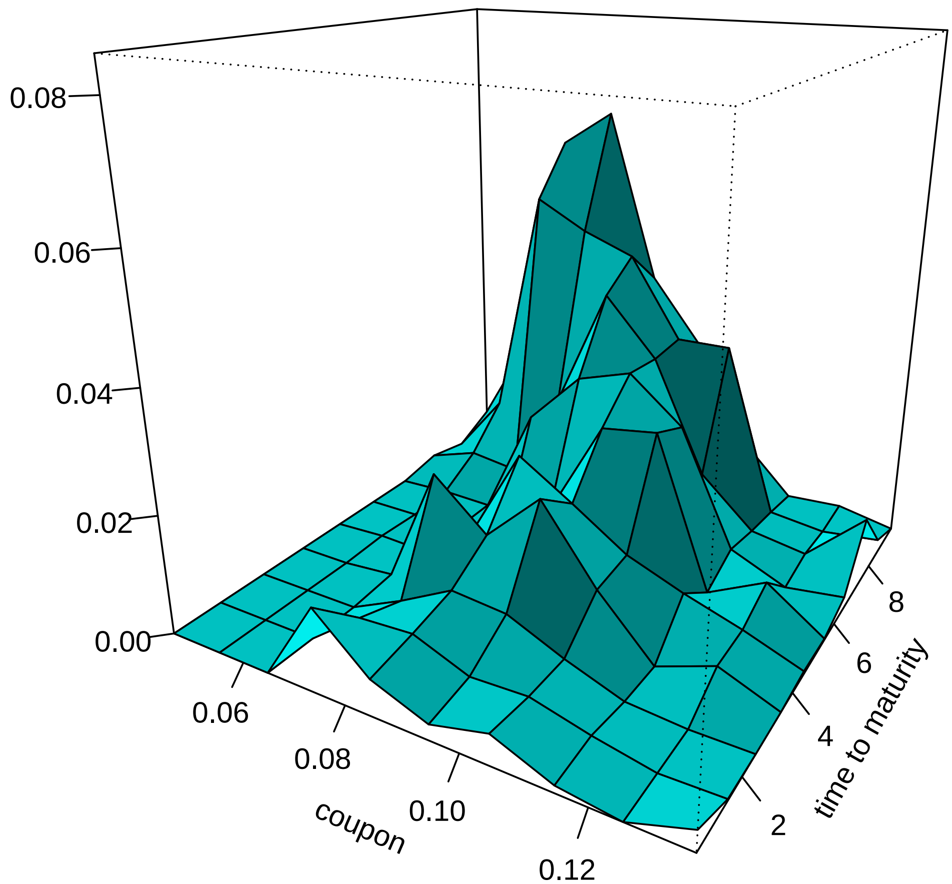


Figure IA-6: November 2004 firms' joint density over coupon and time-to-maturity
The figure displays the November 2004 joint firms' joint density over (i) coupon c and (ii) time-to-maturity T that our model is initialized at.

10-year zero coupon rates **Default intensities**

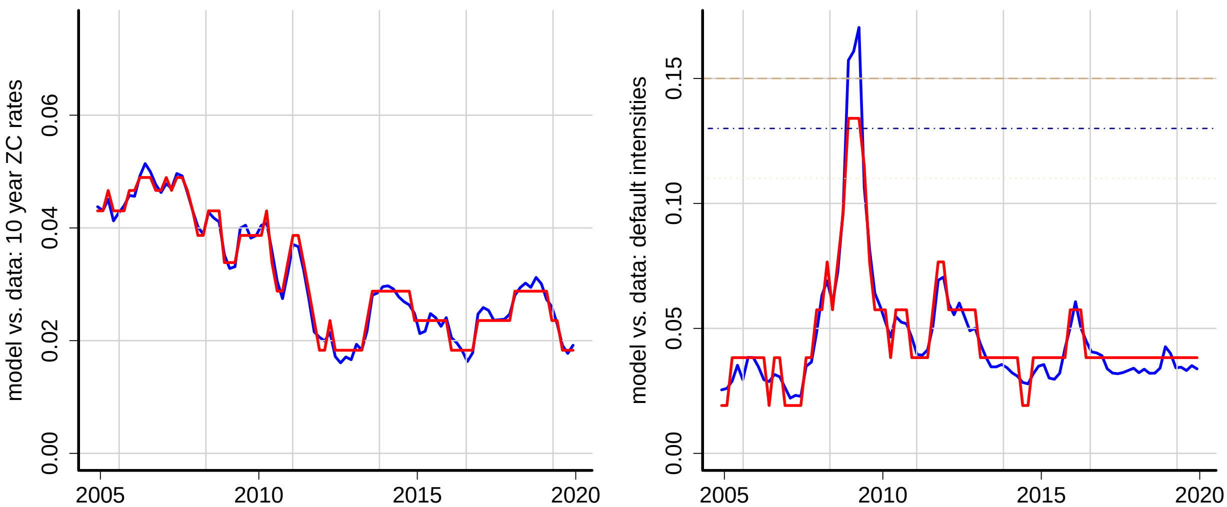


Figure IA-7: Rates and default intensities: model vs. data
This figure shows the 10-year zero coupon rates (left hand side) and the default intensities (right hand side) for the period 2000-2020 in the data as well as fed to our model with a discretized grid, when evaluating the empirical performance of our model for that time period.

Standard deviation of coupon **Standard deviation of time-to-maturity**

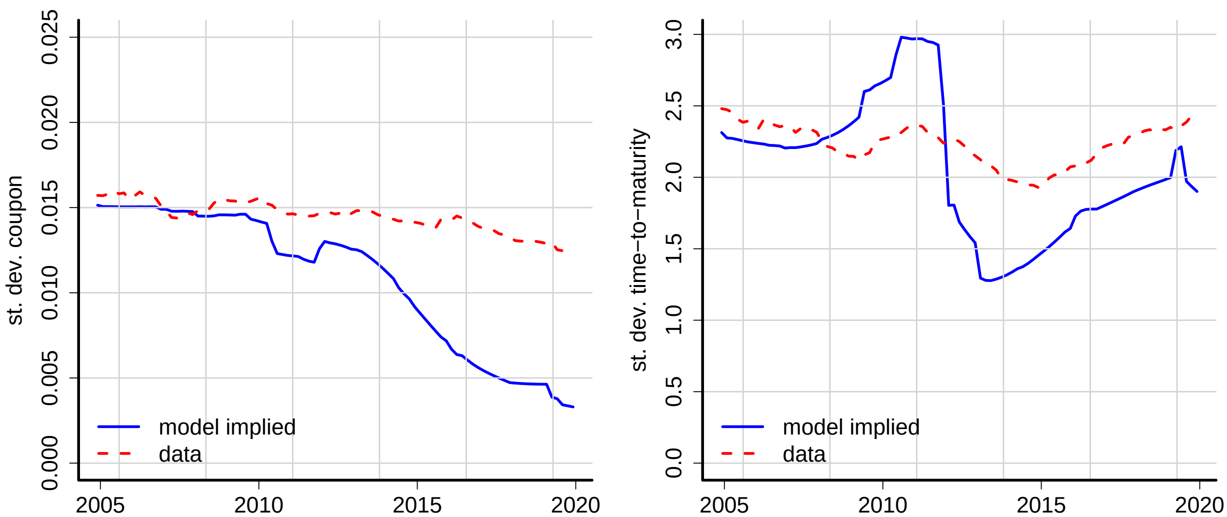


Figure IA-8: Model vs. data: standard deviations
Left (resp. right) hand side shows the firms' standard deviation of coupons (resp. time-to-maturity) in the data (in dash red) and implied by our model (in solid blue) for the time period 2004-2020.

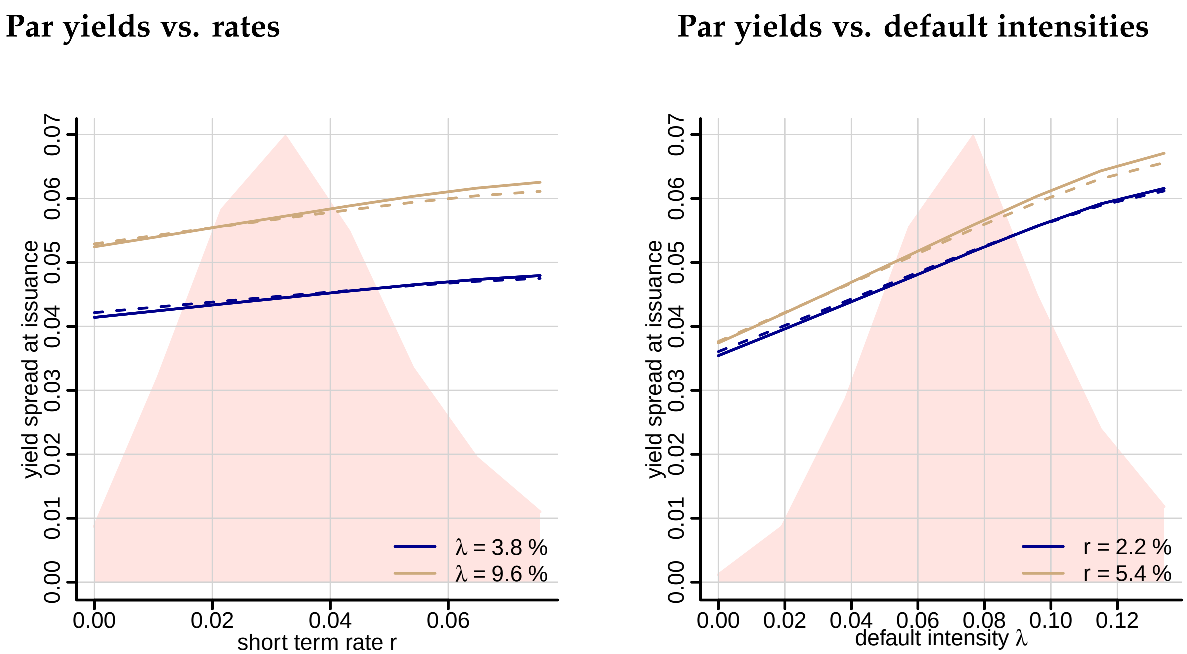
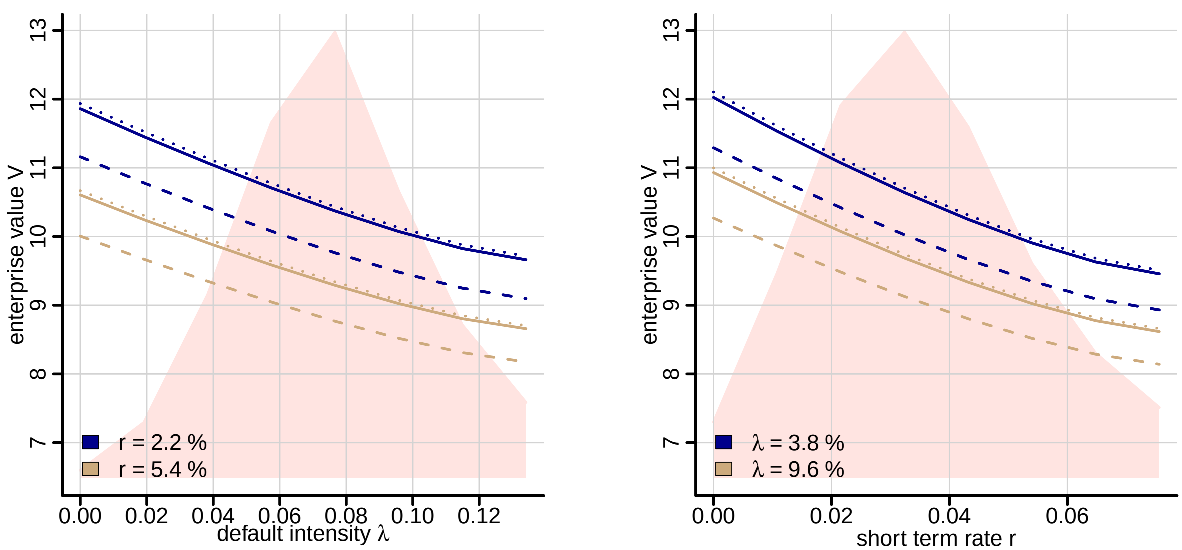


Figure IA-9: Model implied par yields

Both figures show model implied par spreads to the risk-free benchmark for callable (resp. non-callable) debt at the time of issuance in solid blue (resp. dashed red) line. Left hand side depicts those par spreads as a function of the short rate prevalent at the time of debt issuance, while the right hand side depicts those par spreads as a function of default intensities prevalent at such time.

Figure IA-10: Enterprise value with high rollover risk

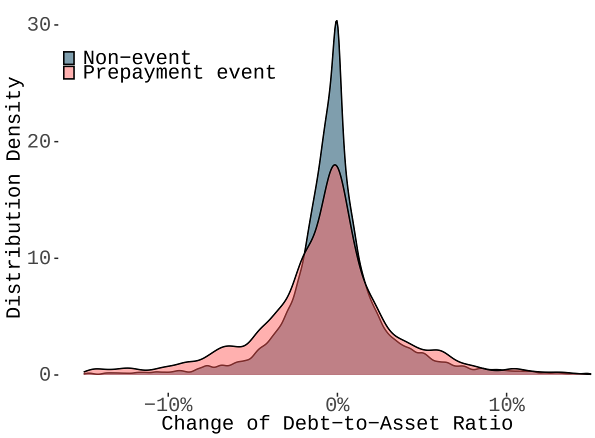
(a): V vs. default intensities **(b):** V vs. interest rates



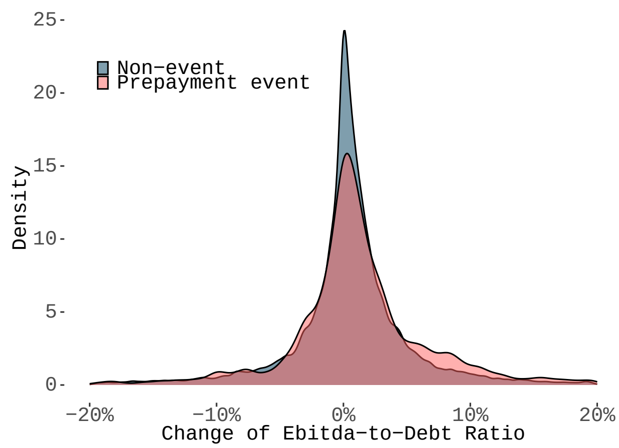
Left hand side shows enterprise value at the time of a new bond issuance as a function of the default intensity λ for various levels of interest rates for the callable debt equilibrium (solid lines), the non-callable debt equilibrum (dashed lines) and the no-rollover risk equilibrium (dot-dashed lines); right hand side instead shows these same enterprise values as a function of the short rate r for various levels of default intensities. These calculations assume the sudden stop occurs anytime default intensities are above $\bar{\lambda} = 11\%$, corresponding to an ergodic average percentage of time spent in the sudden stop state equal to 15%.

Figure IA-11: Leverage changes around prepayment events

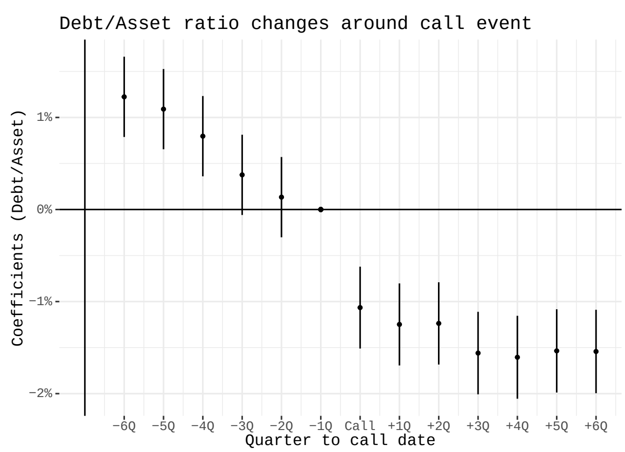
(a): Debt to Asset ratio changes



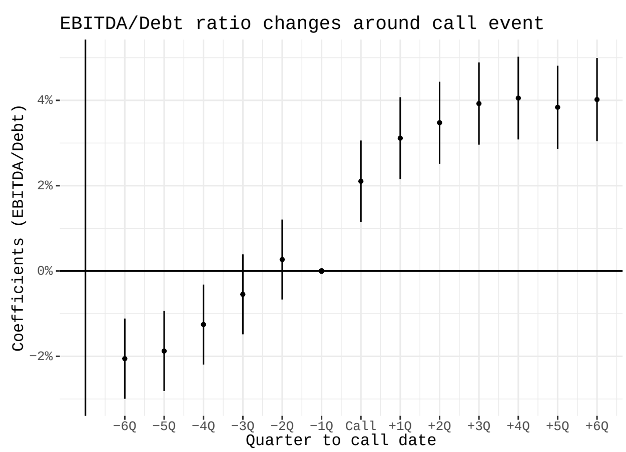
(b): EBITDA to Debt ratio changes



(c): Debt to Asset ratio changes



(d): EBITDA to Debt ratio changes



This figure illustrates leverage changes around prepayment event triggered by call exercise. By average, the Debt to Asset ratio reduces 1.0% around the call event, which is economical insignificant comparing to the sample average of 51.5%. Similarly, the EBITDA to Debt ratio increase around 2.1% around the call event, where the corresponding sample average is 27.0%.

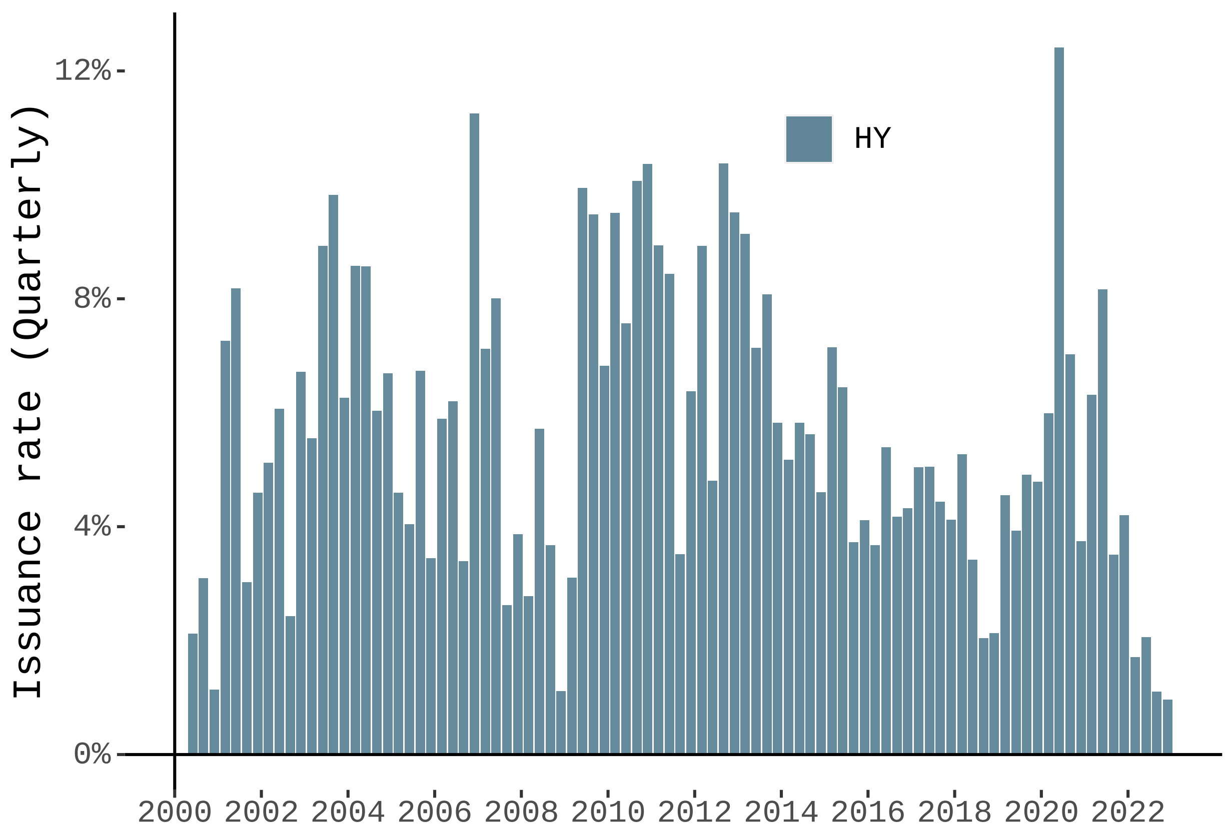


Figure IA-12: Quarterly issuance rate
The figure displays the issuance rate of HY bonds in each quarter from 2000 to 2022. The lowest issuance rate happened at 2000-Q4, 2008-Q4 and 2022-Q4.